Physics by fiziks

Learn Physics in Right Way

CSIR NET-JRF Physical Sciences Paper Feb.-2022

Solution-Mathematical Physics

Be Part of Disciplined Learning

Part B

Q25. The position of a particle in one dimension changes in discrete steps. With each step it moves to the right, however, the length of the step is drawn from a uniform distribution from the interval $\left[\lambda-\frac{1}{2}w,\lambda+\frac{1}{2}w\right]$, where λ and w are positive constants. If X denotes the distance from the starting point after N steps, the standard deviation $\sqrt{\langle X^2\rangle-\langle X\rangle^2}$ for large values of N is

(a) $\frac{\lambda}{2}\times\sqrt{N}$ (b) $\frac{\lambda}{2}\times\sqrt{\frac{N}{3}}$ (c) $\frac{w}{2}\times\sqrt{N}$ (d) $\frac{w}{2}\times\sqrt{\frac{N}{3}}$

Ans. 25: (d)

Solution:

Let $x_1,x_{2,....}x_N$ denote the length. Hence total length of steps $x=x_1+x_2+....+x_N$

$x_1,x_{2,....}x_N$ are random variables from a uniform distribution from the interval $\left[\lambda-\frac{1}{2}w,\lambda+\frac{1}{2}w\right]$

$$Var[x]=Var(x_1+x_2+....+x_N)=\sum_{i=1}^{N}Var(x_i)+2\sum_{i<j}Var(x_i,x_j)$$

$\because x_1,x_{2,....}x_N$ are independent random variables; $\therefore \sum_{i<j}Var(x_i,x_j)=0$

$$\therefore Var[x]=\sum_{i=1}^{N}Var(x_i)=N\frac{\left[\left(\lambda+\frac{1}{2}w\right)-\left(\lambda-\frac{1}{2}w\right)\right]^2}{12}=N\frac{w^2}{12}$$

Hence standard deviation $=\sqrt{Var[x]}=\frac{w}{2}\sqrt{\frac{N}{3}}$

Q26. The volume of the region common to the interiors of two infinitely long cylinders defined by $x^2+y^2=25$ and $x^2+4z^2=25$ is best approximated by

(a) 225 (b) 333 (c) 423 (d) 625

Ans. 26: (b)

Solution:

$$x^2+y^2=25\Rightarrow y=\pm\sqrt{25-x^2};\quad x^2+4z^2=25\Rightarrow z=\pm\frac{\sqrt{25-x^2}}{2}$$

In any of the above equations, `x' varies from -5 to 5. Therefore, the volume bounded in the intersecting region is

$$V=\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}}\int_{-\frac{\sqrt{25-x^2}}{2}}^{\frac{\sqrt{25-x^2}}{2}}dzdydx=8\int_0^5\int_0^{\sqrt{25-x^2}}\int_0^{\frac{\sqrt{25-x^2}}{2}}dzdydx=8\int_0^5\int_0^{\sqrt{25-x^2}}\frac{\sqrt{25-x^2}}{2}dydx$$

$$V=8\int_0^5\frac{\sqrt{25-x^2}}{2}\sqrt{25-x^2}\,dx=4\int_0^5\left(25-x^2\right)dx=4\left|25x-\frac{x^3}{3}\right|_0^5=4\left|125-\frac{125}{3}\right|$$

$V=4\times\frac{250}{3}=333.33$. Hence (b) is correct option.

Q36. A discrete random variable X takes a value from the set $\{-1,0,1,2\}$ with the corresponding probabilities $p(X)=3/10,2/10,2/10$ and $3/10$, respectively. The probability distribution $q(Y)=\left(q(0),q(1),q(4)\right)$ of the random variable $Y=X^2$ is

(a) $\left(\frac{1}{5},\frac{3}{5},\frac{1}{5}\right)$ (b) $\left(\frac{1}{5},\frac{1}{2},\frac{3}{10}\right)$ (c) $\left(\frac{2}{5},\frac{2}{5},\frac{1}{5}\right)$ (d) $\left(\frac{3}{10},\frac{3}{10},\frac{2}{5}\right)$

Ans. 36: (b)

Solution: Given that, $X=\{-1,0,1,2\}$; $p(X)=\{\frac{3}{10},\frac{2}{10},\frac{2}{10},\frac{3}{10}\}$

$X^2=\{0,1,4\}$; $p(X^2)=\{—,—,---\}=?$

For, $X^2=0, X=0\Rightarrow p(X^2=0)=p(X=0)=\frac{2}{10}$

For, $X^2=1, X=\pm1;\ \Rightarrow p(X^2=1)=p(X=1)+p(X=-1)=\frac{3}{10}+\frac{2}{10}=\frac{1}{2}$

For ($X=-2$ is not in the list),

$X^2=4, X=\pm2\quad\Rightarrow p(X^2=4)=p(X=2)=\frac{3}{10}$

Thus, $p(X^2)=\{\frac{1}{5},\frac{1}{2},\frac{3}{10}\}$ Hence, (b) is correct option.

Q41. The equation of motion of a one-dimensional forced harmonic oscillator in the presence of a dissipative force is described by $\frac{d^2x}{dt^2}+10\frac{dx}{dt}+16x=6te^{-8t}+4t^2e^{-2t}$. The general form of the particular solution, in terms of constants A,B etc., is

(a) $t\left(At^2+Bt+C\right)e^{-2t}+\left(Dt+E\right)e^{-8t}$ (b) $\left(At^2+Bt+C\right)e^{-2t}+\left(Dt+E\right)e^{-8t}$

(c) $t\left(At^2+Bt+C\right)e^{-2t}+t\left(Dt+E\right)e^{-8t}$ (d) $\left(At^2+Bt+C\right)e^{-2t}+t\left(Dt+E\right)e^{-8t}$

Ans. 41: (c)

Solution:

Given differential equation is $\frac{d^2x}{dt^2}+10\frac{dx}{dt}+16x=6te^{-8t}+4t^2e^{-2t}$

Auxiliary equation is $D^2+10D+16=0\Rightarrow(D+8)(D+2)=0\Rightarrow D=-8,-2$

Thus, the complementary function can be written as

$y_{cf}=ae^{-8t}+be^{-2t}$.............(1)

$$PI=\frac{1}{D^2+10D+16}6te^{-8t}+4t^2e^{-2t}=\frac{1}{D^2+10D+16}(6te^{-8t})+\frac{1}{D^2+10D+16}(4t^2e^{-2t})\text{.......(2)}$$

$$\frac{1}{D^2+10D+16}(6te^{-8t})=\frac{1}{\cancel{6}}\left[\frac{1}{D+2}-\frac{1}{D+8}\right]\cancel{6}te^{-8t}=\frac{1}{D+2}te^{-8t}-\frac{1}{D+8}te^{-8t}$$

$$=e^{-2t}\int e^{2t}te^{-8t}dt-e^{-8t}\int e^{8t}te^{-8t}dt=e^{-2t}\int te^{-6t}dt-e^{-8t}\int tdt$$

$$=e^{-2t}\left[t\frac{e^{-6t}}{-6}-\int(1)\frac{e^{-6t}}{-6}dt\right]-\frac{t^2}{2}e^{-8t}=e^{-2t}\left[t\frac{e^{-6t}}{-6}-\frac{e^{-6t}}{36}\right]-\frac{t^2}{2}e^{-8t}=t\frac{e^{-8t}}{-6}-\frac{e^{-8t}}{36}-\frac{t^2}{2}e^{-8t}\ =$$

$$=-\frac{t^2}{2}e^{-8t}-\frac{te^{-8t}}{6}-\frac{e^{-8t}}{36}$$

The last terms in the above expression can be coupled with complementary function

Therefore, $A'=t\left[Dt+E\right]e^{-8t}$...............(3)

$$\frac{1}{D^2+10D+16}(4t^2e^{-2t})=\frac{1}{6}\left[\frac{1}{D+2}-\frac{1}{D+8}\right]4te^{-2t}=\frac{2}{3}\left[\frac{1}{D+2}t^2e^{-2t}-\frac{1}{D+8}t^2e^{-2t}\right]$$

$$=\frac{2}{3}\left[e^{-2t}\int t^2dt-e^{-8t}\int t^2e^{6t}dt\right]=\frac{2}{3}\left[e^{-2t}\frac{t^3}{3}-e^{-8t}\left[\frac{t^2e^{6t}}{6}-\int 2t\frac{e^{6t}}{6}dt\right]\right]$$

$$=\frac{2}{3}\left[e^{-2t}\frac{t^3}{3}-e^{-8t}\left[\frac{t^2e^{6t}}{6}-\frac{1}{3}\int te^{6t}dt\right]\right]=\frac{2}{3}\left[e^{-2t}\frac{t^3}{3}-e^{-8t}\left[\frac{t^2e^{6t}}{6}-\frac{1}{3}\left[\frac{te^{6t}}{6}-\int\frac{e^{6t}}{6}dt\right]\right]\right]$$

$$=\frac{2}{3}\left[e^{-2t}\frac{t^3}{3}-e^{-8t}\left[\frac{t^2e^{6t}}{6}-\frac{1}{3}\left[\frac{te^{6t}}{6}-\int\frac{e^{6t}}{6}dt\right]\right]\right]=\frac{2}{3}\left[e^{-2t}\frac{t^3}{3}-e^{-8t}\left[\frac{t^2e^{6t}}{6}-\frac{1}{3}\left[\frac{te^{6t}}{6}-\frac{e^{6t}}{36}\right]\right]\right]$$

$$=\frac{2}{3}\left[e^{-2t}\frac{t^3}{3}-e^{-8t}\left[\frac{t^2e^{6t}}{6}-\frac{te^{6t}}{18}+\frac{e^{6t}}{108}\right]\right]=\frac{2}{3}\left[e^{-2t}\frac{t^3}{3}-\frac{t^2e^{-2t}}{6}+\frac{te^{-2t}}{18}-\frac{e^{-2t}}{108}\right]$$

$$=\frac{2t^3e^{-2t}}{9}-\frac{t^2e^{-2t}}{9}+\frac{te^{-2t}}{27}-\frac{e^{-2t}}{162}$$

The last terms in the above expression can be coupled with complementary function

Therefore, $B'=t\left[At^2+BT+C\right]e^{-2t}$...........(4)

From (3) and (4); $PI=t\left[At^2+BT+C\right]e^{-2t}+t\left[Dt+E\right]e^{-8t}$............(5)

Thus, (c) is correct option.

Q44. A generic 3×3 real matrix A has eigenvalues $0,1$ and 6, and I is the 3×3 identity matrix. The quantity/quantities that cannot be determined from this information is/are the

(a) eigenvalue of $(I+A)^{-1}$ (b) eigenvalue of $(I+A^TA)$

(c) determinant of A^TA (d) rank of A

Ans. 44: (b)

Solution: Given Eigen values are $=0,1,6$

Eigen values of $I+A$ are $=1+$ Eigen values of A

Therefore, Eigen values of $(I+A)^{-1}$ are $=1,\frac{1}{2},\frac{1}{7}$

Therefore, `a' can be determined

$|A|=0\times1\times6=0$, $|A^T|=0\times1\times6=0$. Therefore, $|AA^T|=|A||A^T|=0$

Thus, (c) can also be determined

As one Eigen value is 0. Therefore, rank is 3-1=2. Hence, `d' can also be determined.

Thus, `b' cannot be determined. Hence, it is the correct answer.

Q45. The volume integral $I=\iiint_V \vec{A}\cdot\left(\vec{\nabla}\times\vec{A}\right)d^3x$, is over a region V bounded by a surface Σ (an infinitesimal area element being $\hat{n}ds$, where $\hat{n}$ is the outward unit normal). If it changes to $I+\Delta I$ when the vector $\vec{A}$ is changed to $\vec{A}+\vec{\nabla}\Lambda$, then ΔI can be expressed as

(a) $\iiint_V \vec{\nabla}\cdot\left(\vec{\nabla}\Lambda\times\vec{A}\right)d^3x$ (b) $-\iiint_V \nabla^2\Lambda\, d^3x$

(c) $-\oiint_\Sigma \left(\vec{\nabla}\Lambda\times\vec{A}\right).\hat{n}ds$ (d) $\oiint_\Sigma \vec{\nabla}\Lambda.\hat{n}ds$

Ans. 45: (c)

Solution:

$$I=\int\int\int\vec{A}\cdot(\vec{\nabla}\times\vec{A})d^3x$$

$$I+\Delta I=\int\int\int(\vec{A}+\vec{\nabla}\lambda)\cdot(\vec{\nabla}\times(\vec{A}+\vec{\nabla}\lambda))d^3x$$

$$I+\Delta I=\int\int\int(\vec{A}+\vec{\nabla}\lambda)\cdot(\vec{\nabla}\times(\vec{A})+\vec{\nabla}\times(\vec{\nabla}\lambda))d^3x$$

Now, the curl of the gradient always vanishes.

Therefore, the above equation becomes.

$$I+\Delta I=\int\int\int\left(\vec{A}\cdot\left(\vec{\nabla}\times\vec{A}\right)+\vec{\nabla}\lambda\cdot\left(\vec{\nabla}\times\vec{A}\right)\right)d^3x$$

$$I+\Delta I=\int\int\int\vec{A}\cdot\left(\vec{\nabla}\times\vec{A}\right)d^3x+\int\int\int\vec{\nabla}\lambda\cdot\left(\vec{\nabla}\times\vec{A}\right)d^3x$$

The first term in the above expression is just the I. Thus, we get

$\Delta I = \iiint \vec{\nabla}\lambda \cdot \left(\vec{\nabla}\times\vec{A}\right) d^3x ....(A)$

We know, $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}\cdot\left(\vec{\nabla}\times\vec{B}\right)$

Using, $\vec{A}=\vec{A}, \quad \vec{B}=\vec{\nabla}\lambda$ in the above expression, we get

$\vec{\nabla}\cdot(\vec{A}\times\vec{\nabla}\lambda) = \vec{\nabla}\lambda\cdot\left(\vec{\nabla}\times\vec{A}\right) - \vec{A}\cdot\left(\vec{\nabla}\times\vec{\nabla}\lambda\right)$

The second term vanishes again. Therefore, we get $\vec{\nabla}\cdot(\vec{A}\times\vec{\nabla}\lambda) = \vec{\nabla}\lambda\cdot\left(\vec{\nabla}\times\vec{A}\right)$

Substituting, this result in (A), we get $\Delta I = \iiint \vec{\nabla}\cdot(\vec{A}\times\vec{\nabla}\lambda) d^3x$

Using, divergence theorem, we get $\Delta I = \oiint (\vec{A}\times\vec{\nabla}\lambda)\cdot\hat{n}ds = -\oiint (\vec{\nabla}\lambda\times\vec{A})\cdot\hat{n}ds$

Thus, (c) is the correct option.

Part C

Q46. The Newton-Raphson method is to be used to determine the reciprocal of the number $x=4$. If we start with the initial guess 0.20 then after the first iteration the reciprocal is

(a) 0.23 (b) 0.24 (c) 0.25 (d) 0.26

Ans. 46: (b)

Solution: To find the inverse of 4, let $x=\frac{1}{4} \Rightarrow f(x)=\frac{1}{x}-4=0$

Thus, we need the solution of this equation after first iteration.

Starting point, $x_0 = 0.20$

$$\Rightarrow f(x_0)=\frac{1}{0.20}-4=5-4=1; \quad f'(x_0)=-\frac{1}{x^2}\bigg|_{x=x_0}=-\frac{1}{(0.20)^2}=-25$$

$$x_{n+1}=x_n-\frac{f(x_0)}{f'(x_0)}=0.20-\frac{1}{-25}=0.20+0.04=0.24$$

Hence, (b) is correct option.

Q67. The Legendre polynomials $P_n(x), n=0,1,2,...,$ satisfying the orthogonality condition $\int_{-1}^{1} P_n(x)\, p_m(x)\, dx=\frac{2}{2n+1}\delta_{nm}$ on the interval $[-1,+1]$, may be defined by the Rodrigues formula $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$. The value of the definite integral $\int_{-1}^{1}\left(4+2x-3x^2+4x^3\right)P_3(x)\,dx$ is

(a) $3/5$ (b) $11/15$ (c) $23/32$ (d) $16/35$

Ans. 67: (d)

Solution:

Let

$$4+2x-3x^2+4x^3=a_0P_0(x)+a_1P_1(x)+a_2P_2(x)+a_3P_3(x)$$

$$4+2x-3x^2+4x^3=a_0+a_1x+a_2\left[\frac{3x^2-1}{2}\right]+a_3\left[\frac{5x^3-3x}{2}\right]=\left[a_0-\frac{a_2}{2}\right]+\left[a_1-\frac{3a_3}{2}\right]x+\frac{3}{2}a_2x^2+\frac{5a_3x^3}{2}$$

Comparing, we get

$$a_2\frac{3}{2}=-3\Rightarrow a_2=-2,\frac{5}{2}a_3=4\Rightarrow a_3=\frac{8}{5}$$

$$a_0-\frac{a_2}{2}=4\Rightarrow a_0=4+\frac{-2}{2}=3$$

$$a_1-\frac{3a_3}{2}=2\Rightarrow a_1=\frac{24}{10}+2=\frac{12}{5}+2=\frac{22}{5}$$

Therefore,

$$\int_{-1}^{1}(4+2x-3x^2+4x^3)P_3(x)dx=\int_{-1}^{1}\left(3P_0(x)+\frac{22}{5}P_1(x)-2P_2(x)+\frac{8}{5}P_3(x)\right)P_3(x)dx=$$

$$\int_{-1}^{1}\frac{8}{5}P_3(x)P_3(x)dx=\frac{8}{5}\int_{-1}^{1}P_3(x)P_3(x)dx=\frac{8}{5}\frac{2}{2\times3+1}=\frac{16}{35}$$

Other integrals vanish because of orthogonal property. Thus, (d) is correct option.

Q68. If we use the Fourier transform $\phi(x,y)=\int e^{ikx}\phi_k(y)dk$ to solve the partial differential equation $-\frac{\partial^2\phi(x,y)}{\partial y^2}-\frac{1}{y^2}\frac{\partial^2\phi(x,y)}{\partial x^2}+\frac{m^2}{y^2}\phi(x,y)=0$ in the half-plane $\{(x,y):-\infty<x<\infty,0<y<\infty\}$ the Fourier modes $\phi_k(y)$ depend on y as y^α and y^β. The value of α and β are

(a) $\frac{1}{2}+\sqrt{1+4(k^2+m^2)}$ and $\frac{1}{2}-\sqrt{1+4(k^2+m^2)}$

(b) $1+\sqrt{1+4(k^2+m^2)}$ and $1-\sqrt{1+4(k^2+m^2)}$

(c) $\frac{1}{2}+\frac{1}{2}\sqrt{1+4(k^2+m^2)}$ and $\frac{1}{2}-\frac{1}{2}\sqrt{1+4(k^2+m^2)}$

(d) $1+\frac{1}{2}\sqrt{1+4(k^2+m^2)}$ and $1-\frac{1}{2}\sqrt{1+4(k^2+m^2)}$

Ans. 68: (c)

Solution:

$$\phi(x,y)=\int e^{ikx}\phi_k(y)dk \Rightarrow \phi_k(y)=\int e^{-ikx}\phi(x,y)dx$$

Given equation

$$-\frac{\partial^2\phi(x,y)}{\partial y^2}-\frac{1}{y^2}\frac{\partial^2\phi(x,y)}{\partial x^2}+\frac{m^2}{y^2}\phi(x,y)=0\Rightarrow -y^2\frac{\partial^2\phi(x,y)}{\partial y^2}-\frac{\partial^2\phi(x,y)}{\partial x^2}+m^2\phi(x,y)=0$$

Multiplying both sides by e^{-ikx} and integrating with respect to `x', we get

$$-y^2\frac{\partial^2}{\partial y^2}\left[\int e^{-ikx}\phi(x,y)dx\right]-\int e^{-ikx}\frac{\partial^2\phi(x,y)}{\partial x^2}dx+m^2\int e^{-ikx}\phi(x,y)dx=0............(1)$$

Using $\int e^{-ikx}\frac{\partial^2\phi(x,y)}{\partial x^2}dx=(-ik)^2\int e^{-ikx}\phi(x,y)dx$ in (1), we get

$$-y^2\frac{\partial^2\phi_k(y)}{\partial y^2}-(-ik)^2\phi_k(y)+m^2\phi_k(y)=0$$

$$\Rightarrow y^2\frac{\partial^2\phi_k(y)}{\partial y^2}-(k^2+m^2)\phi_k(y)=0..............(2)$$

Using, $y=e^z$, (2) becomes $\left[D(D-1)-(k^2+m^2)\right]\phi_k(z)=0...........(3)$

The auxiliary equation can be written as $D^2-D-(k^2+m^2)=0\Rightarrow D=\frac{1\pm\sqrt{1+4(k^2+m^2)}}{2}$

The solution of (3) can therefore be written as

$$\phi_k(z)=ae^{\frac{1+\sqrt{1+4(k^2+m^2)}}{2}z}+be^{\frac{1-\sqrt{1+4(k^2+m^2)}}{2}z}\Rightarrow \phi_k(z)=a(e^z)^{\frac{1+\sqrt{1+4(k^2+m^2)}}{2}}+b(e^z)^{\frac{1-\sqrt{1+4(k^2+m^2)}}{2}}$$

Reverting to original variable, we get $\phi_k(y)=a(y)^{\frac{1+\sqrt{1+4(k^2+m^2)}}{2}}+b(y)^{\frac{1-\sqrt{1+4(k^2+m^2)}}{2}}$

Therefore, $\alpha=\frac{1+\sqrt{1+4(k^2+m^2)}}{2}$ and $\beta=\frac{1-\sqrt{1+4(k^2+m^2)}}{2}$ Thus, (c) is correct option.

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Solution-Classical Mechanics

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Part B

Q22. A particle in one dimension executes oscillatory motion in a potential $V(x) = A|x|$, where $A > 0$ is a constant of appropriate dimension. If the time period T of its oscillation depends on the total energy E as E^a, then the value of a is

(a) $1/3$ (b) $1/2$ (c) $2/3$ (d) $3/4$

Ans. 22: (b)

Solution: Total energy $E = \frac{p^2}{2m} + A|x|$

Action angle variable $J = 4\int_0^{E/A} \sqrt{2m(E - A|x|)}dx = 4\sqrt{2mE}\int_0^{E/A} \sqrt{1 - \frac{A}{E}x}\, dx$

For $0 \le x \le \frac{E}{A} \quad \to |x| = x$

Let $\frac{A}{E}x = t \to dx = \frac{E}{A}dt \quad ; J = 4\sqrt{2mE}\,\frac{E}{A}\int_0^1 \sqrt{1-t}dt \Rightarrow J = x_0 E^{3/2}$

Time period $T = \frac{\partial J}{\partial E} = \frac{3}{2}x_0 E^{1/2} \Rightarrow \alpha = \frac{1}{2}$

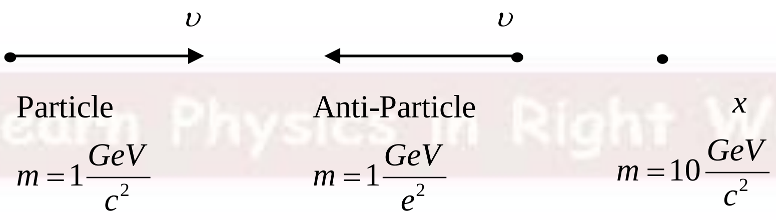
Q24. A particle of mass $1GeV/c^2$ and its antiparticle, both moving with the same speed v, produce new particle x of mass $10GeV/c^2$ in a head on collision. The minimum value of v required for this process is closest to

(a) $0.83c$ (b) $0.93c$

(c) $0.98c$ (d) $0.88c$

Ans. 24: (c)

Solution:



Conservation of energy $\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = Mc^2 \Rightarrow \frac{2m\cancel{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = M\cancel{c^2}$

$2 \times 1 = 10\sqrt{1-\frac{v^2}{c^2}} \Rightarrow \frac{1}{25} = 1 - \frac{v^2}{c^2} \Rightarrow v = \frac{\sqrt{24}}{5}c = 0.93c$

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Q29. A monochromatic source emitting radiation with a certain frequency moves with a velocity v away from a stationary observer A. It is moving towards another observer B (also at rest) along a line joining the two. The frequencies of the radiation recorded by A and B are V_A and V_B, respectively. If the ratio $\frac{V_B}{V_A}=7$, then the value of v/c is

(a) $1/2$ (b) $1/4$ (c) $3/4$ (d) $\sqrt{3}/2$

Ans. 29: (c)

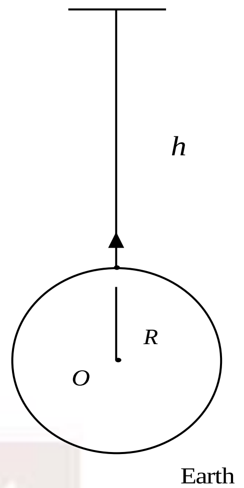
Solution.

$$\upsilon_A=\upsilon_A\sqrt{\frac{c-\upsilon}{c+\upsilon}};\ \upsilon_B=\upsilon_0\sqrt{\frac{c+\upsilon}{c-\upsilon}};\quad \frac{\upsilon_B}{\upsilon_A}=\frac{c+\upsilon}{c-\upsilon}=\frac{1+\upsilon/c}{1-\upsilon/c}=7$$

$$7-7\frac{\upsilon}{c}=1+\frac{\upsilon}{c}\Rightarrow 8\frac{\upsilon}{c}=6\Rightarrow \boxed{\frac{\upsilon}{c}=\frac{3}{4}}$$

Q30. A particle, thrown with a speed v from the earth's surface, attains a maximum height h (measured from the surface of the earth). If v is half the escape velocity and R denotes the radius of earth, then h/R is

(a) $2/3$ (b) $1/3$ (c) $1/4$ (d) $1/2$



Ans. 30: (b)

Solution.: $\upsilon=\frac{1}{2}\upsilon_e=\frac{1}{2}\sqrt{\frac{2GM}{R}}$. Here M is the mass of the earth.

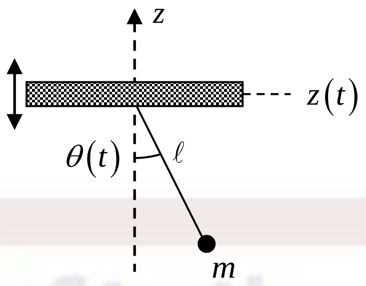
Conservation of mechanical energy

$$-\frac{GMm}{R}+\frac{1}{2}m\upsilon^2=-\frac{GMm}{(R+h)}+0\Rightarrow -\frac{GMm}{R}+\frac{GMm}{4R}=-\frac{GMm}{R+h}$$

$$\Rightarrow -\frac{3GMm}{4R}=-\frac{GMm}{R+h}\Rightarrow 3R+3h=4R\Rightarrow 3h=R\ \Rightarrow \boxed{\frac{h}{R}=\frac{1}{3}}$$

Part C

Q48. The fulcrum of a simple pendulum (consisting of a particle of mass m attached to the support by a massless string of length ℓ) oscillates vertically as $\sin z(t) = a \sin \omega t$, where ω is a constant. The pendulum moves in a vertical plane and $\theta(t)$ denotes its angular position with respect to the z-axis



If $\ell \dfrac{d^2\theta}{dt^2} + \sin\theta\left(g - f(t)\right) = 0$ (where g is the acceleration due to gravity) describes the equation of motion of the mass, then $f(t)$ is

(a) $a\omega^2 \cos \omega t$ (b) $a\omega^2 \sin \omega t$ (c) $-a\omega^2 \cos \omega t$ (d) $-a\omega^2 \sin \omega t$

Ans. 48: (b)

Solution:

$x = l\sin\theta$, $z = l\cos\theta + z(t) = l\cos\theta + a\sin\omega t \Rightarrow x = l\cos\theta\dot{\theta}$, $\dot{z} = -l\sin\theta\dot{\theta} + a\omega\cos\omega t$

$$L = T - V = \frac{1}{2}m\left[\dot{x}^2 + \dot{z}^2\right] - (-mgz)$$

$$L = \frac{1}{2}m\left[l^2\dot{\theta}^2 + a^2\omega^2\cos^2\omega t + 2al\omega\sin\theta\cos\omega t\dot{\theta}\right] + mg\left(l\cos\theta + a\sin\omega t\right)$$

$$\Rightarrow \frac{\partial L}{\partial\dot{\theta}} = ml^2\dot{\theta} + mal\omega\sin\theta\cos\omega t \text{ and } \frac{\partial L}{\partial\theta} = mal\omega\cos\theta\cos\omega t\dot{\theta} - mgl\sin\theta$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) = ml^2\ddot{\theta} + mal\omega\cos\theta\cos\omega t\dot{\theta} - mal\omega^2\sin\theta\sin\omega t$$

$$\because \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = 0 \Rightarrow ml^2\ddot{\theta} - mal\omega^2\sin\theta\sin\omega t + mgl\sin\theta = 0$$

$$\Rightarrow l\frac{d^2\theta}{dt^2} + \sin\theta\left[g - a\omega^2\sin\omega t\right] = 0 \Rightarrow l\frac{d^2\theta}{dt^2} + \sin\theta\left[g - f(t)\right] = 0$$

where $f(t) = a\omega^2\sin\omega t$

Q57. A particle in two dimensions is found to trace an orbit $r(\theta)=r_0\theta^2$. If it is moving under the influence of a central potential $V(r)=c_1r^{-a}+c_2r^{-b}$, where r_0, c_1 and c_2 are constants of appropriate dimensions, the values of a and b, respectively, are

(a) 2 and 4 (b) 2 and 3 (c) 3 and 4 (d) 1 and 3

Ans. 57: (b)

Solution: $u=\frac{1}{r}=\frac{1}{r_0\theta^2}\Rightarrow\frac{du}{d\theta}=-\frac{2}{r_0\theta^3}\Rightarrow\frac{d^2u}{d\theta^2}=\frac{6}{r_0\theta^4}$

Differential equation of the orbit $\frac{d^2u}{d\theta^2}+u=-\frac{m}{\ell^2u^2}f\left(\frac{1}{u}\right)\Rightarrow\frac{6}{r_0\theta^4}+\frac{1}{r_0\theta^2}=-\frac{mr_0^2\theta^4}{\ell^2}f\left(\frac{1}{u}\right)$

$f\left(\frac{1}{u}\right)=-\frac{6\ell^2}{mr_0^3\theta^8}-\frac{\ell^2}{mr_0^3\theta^6}\quad\Rightarrow f(r)=-Ar^{-4}-Br^{-3}$ where A and B are constants

$A=\frac{6\ell^2r_0}{m}$ and $B=\frac{\ell^2}{m}$

$V(r)=-\int f(r)dr=\int\left[Ar^{-4}+Br^{-3}\right]dr=A\frac{r^{-4+1}}{-3}+B\frac{r^{-3+1}}{-2}$

$V(r)=c_1r^{-3}+c_2r^{-2}=c_1r^{-a}+c_2r^{-b}\qquad\Rightarrow a=3,\ b=2$

Q58. A particle of mass m moves in a potential that is $V=\frac{1}{2}m\left(\omega_1^2x^2+\omega_2^2y^2+\omega_3^2z^2\right)$ in the coordinates of a non-inertial frame F. The frame F is rotating with respect to an inertial frame with an angular velocity $\hat{k}\Omega$, where $\hat{k}$ it is the unit vector along their common z-axis. The motion of the particle is unstable for all angular frequencies satisfying

(a) $\left(\Omega^2-\omega_1^2\right)\left(\Omega^2-\omega_2^2\right)>0$ (b) $\left(\Omega^2-\omega_1^2\right)\left(\Omega^2-\omega_2^2\right)<0$

(c) $\left(\Omega^2-(\omega_1+\omega_2)^2\right)\left(\Omega^2-|\omega_1-\omega_2|^2\right)>0$ (d) $\left(\Omega^2-(\omega_1+\omega_2)^2\right)\left(\Omega^2-|\omega_1-\omega_2|^2\right)<0$

Ans. 58: (b)

Q60. A satellite of mass m orbits around earth in an elliptic trajectory of semi-major axis a. At a radial distance $r=r_0$, measured from the centre of the earth, the kinetic energy is equal to half the magnitude of the total energy. If M denotes the mass of the earth and the total energy is $-\frac{GMm}{2a}$, the value of r_0/a is nearest to

(a) 1.33 (b) 1.48 (c) 1.25 (d) 1.67

Ans. 60: (a)

Solution: $TE = -\frac{GMm}{2a}, \quad KE = \frac{1}{2}|TE| = \frac{GMm}{4a}$

$$PE = TE - KE = -\frac{GMm}{2a} - \frac{GMm}{4a} \quad \Rightarrow PE = -\frac{3GMm}{4a} \qquad \ldots\ldots\ldots\ldots(1)$$

The potential energy at $r = r_0$ will be $PE = -\frac{GMm}{r_0}$ $\qquad \ldots\ldots\ldots\ldots(2)$

From Eqs. (1) and (2); $-\frac{3GMm}{4a} = -\frac{GMm}{r_0} \Rightarrow \frac{r_0}{a} = \frac{4}{3} = 1.33$

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Solution-Electromagnetic Theory

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Part B

Q23. The components of the electric field, in a region of space devoid of any change or current sources, are given to be $E_i = a_i + \sum_{j=1,2,3} b_{ij} x_j$, where a_i and b_{ij} are constants independent of the coordinates. The number of independent components of the matrix b_{ij} is

(a) 5 (b) 6 (c) 3 (d) 4

Ans. 23: (a)

Solution.: $E_i = a_i + \sum_{j=1}^{3} b_{ij} x_j$ .This equation represents a set of three equations

$$\left.\begin{aligned} E_1 &= a_1 + b_{11}x_1 + b_{12}x_2 + b_{13}x_3 \\ E_2 &= a_2 + b_{21}x_1 + b_{22}x_2 + b_{23}x_3 \\ E_3 &= a_3 + b_{31}x_1 + b_{32}x_2 + b_{33}x_3 \end{aligned}\right] -(1)$$

Let $E_1 = E_x, E_2 = E_y, E_3 = E_z$ and $x_1 = x, x_2 = y, x_3 = z, (1)$ can be written as

$$\left.\begin{aligned} E_x &= a_1 + b_{11}x + b_{12}y + b_{13}z \\ E_y &= a_2 + b_{21}x + b_{22}y + b_{23}z \\ E_z &= a_3 + b_{31}x + b_{32}y + b_{33}z \end{aligned}\right] -(2)$$

For electrostatic field, $\vec{\nabla} \times \vec{E} = 0 \Rightarrow \begin{vmatrix} x & y & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = 0$

Let's just look at the x-component (which will be equal to zero). $\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = 0 - (3)$

Using (2); $\frac{\partial E_z}{\partial y} = b_{32}, \frac{\partial E_y}{\partial z} = b_{23}$. From (3); $b_{32} - b_{23} = 0 \Rightarrow b_{32} = b_{23}$

Similarly, we will get from 'y' and 'z' components $b_{13} = b_{31}, b_{21} = b_{12}$

Thus, it means b_{ij} is symmetric. A symmetric matrix will have,

3 diagonal +3 off- diagonal = 6-independent components $-(4)$

Also, as the region is charge free, therefore $\vec{\nabla} \cdot \vec{E} = 0 \Rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 - (5)$

From (2), $\frac{\partial E_x}{\partial x} = b_{11}, \frac{\partial E_y}{\partial y} = b_{22}, \frac{\partial E_z}{\partial z} = b_{33}$. Putting, this result in (5), we get

$b_{11} + b_{22} + b_{33} = 0$. This implies that atleast one of the diagonal elements is dependent. Therefore, the total number of independent components = 6-1=5. Therefore, (a) is correct option.

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Q37. In an experiment to measure the charge to mass ratio e/m of the electron by Thomson's method, the values of the deflecting electric field and the accelerating potential are $6\times10^6 N/C$ (newton per coulomb) and $150V$, respectively. The magnitude of the magnetic field that leads to zero deflection of the electron beam is closest to

(a) $0.6T$ (b) $1.2T$ (c) $0.4T$ (d) $0.8T$

Ans. 37: (d)

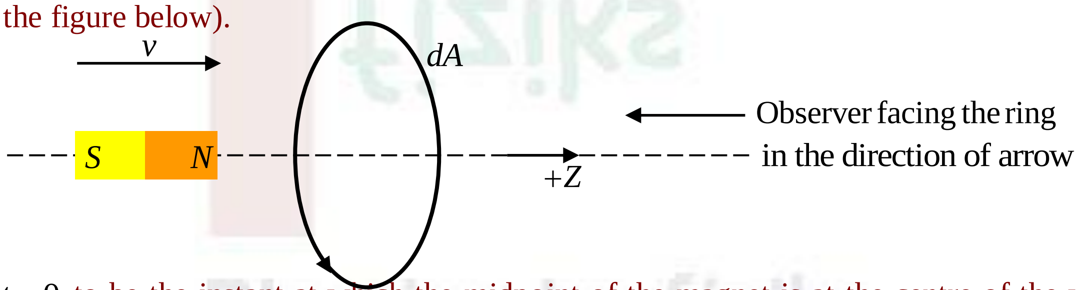
Solution.: Let's determine the velocity of an electron accelerated to 150V.

Using, the classical formula relating kinetic energy and accelerating potential,

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2\times150\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 7.26\times10^6 m/s$$

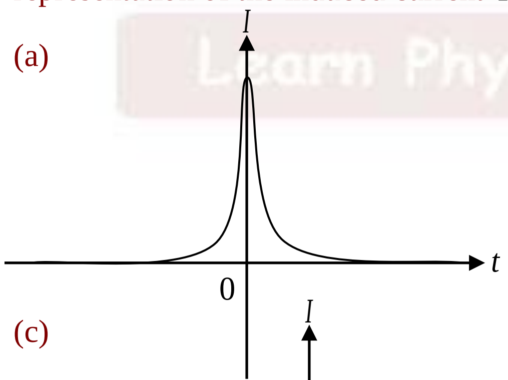
For, zero deflection, $B\cancel{e}v = \cancel{e}V \Rightarrow B = \frac{E}{v} = \frac{6\times10^6}{7.26\times10^6} \cong 0.83T$. Thus, (d) is the correct option.

Q39. A conducting wire in the shape of a circle lies on the (x, y)-plane with its centre at the origin. A bar magnet moves with a constant velocity towards the wire along the z -axis (as shown in the figure below).

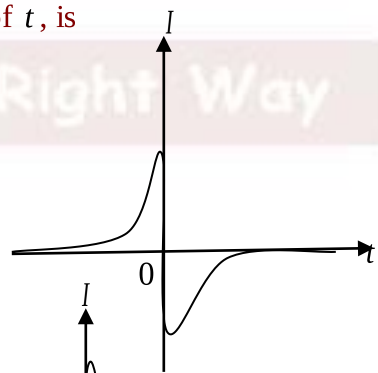


We take $t = 0$ to be the instant at which the midpoint of the magnet is at the centre of the wire loop and the induced current to be positive when it is counter-clockwise as viewed by the observer facing the loop and the incoming magnet. In these conventions, the best schematic representation of the induced current $I(t)$ as a function of t, is

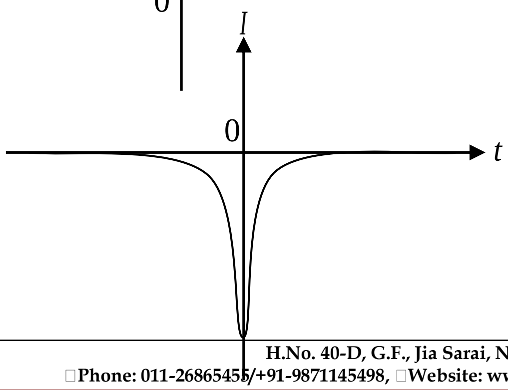
(a)



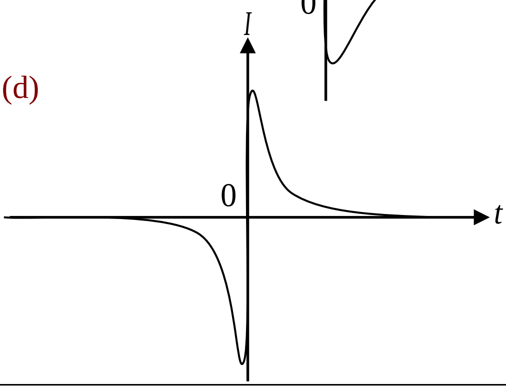
(b)



(c)

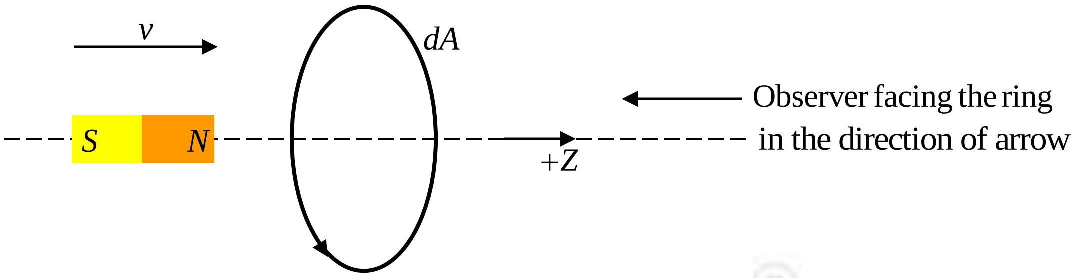


(d)



Ans. 39: (d)

Solution: As, north pole of the magnet moves towards the coil, the induced current must flow in a direction (clockwise as seen from right) to create north polarity on left. However, the current seen from right and flowing counterclockwise is to be considered positive.



This current will produce a south polarity on the left of the coil. Thus, as seen by an observer from right, the current must flow clockwise to produce a north polarity on left. This clockwise current will be negative. Thus, as the bar magnet approaches the coil, first induced current will be negative and after it is about to cross, induced current must be positive. Thus, option (d) should be the correct answer.

Q40. The vector potential for an almost point like magnetic dipole located at the origin is $\vec{A} = \frac{\mu \sin\theta}{4\pi r^2}\hat{\phi}$ where (r, θ, ϕ) denote the spherical polar coordinates and $\hat{\phi}$ is the unit vector along $\hat{\phi}$. A particle of mass m and charge q, moving in the equatorial plane of the dipole, starts at time $= t = 0$ with an initial speed v_0 and an impact parameter b. Its instantaneous speed at the point of closest approach is

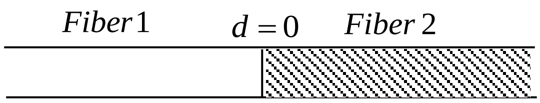
(a) v_0 (b) $0/0$ (c) $v_0 + \frac{\mu q}{4\pi m b^2}$ (d) $\sqrt{v_0^2 + \left(\frac{\mu q}{4\pi m b^2}\right)^2}$

Ans. 40: (a)

Solution: A static magnetic field does not alter the magnitude of speed of a charged particle. It only alters the direction of motion. Hence, its speed will be the same as the one it started with. (i.e., v_0). Thus, (a) is the correct answer.

Part C

Q47. A laser beam propagates from fiber 1 to fiber 2 in a cavity made up of two optical fibers (as shown in the figure). The loss factor of fiber 2 is $10\,dB/km$.



If $E_2(d)$ denotes the magnitude of the electric field in fiber 2 at a distance d from the interface, the ratio $E_2(0)/E_2(d)$ for $d=10\,km$, is

(a) 10^2 (b) 10^3 (c) 10^5 (d) 10^7

Ans. 47: (c)

Solution.: As attenuation leads to a loss of power along the fiber, the output power is significantly less than the couples power. Let the couples optical power is $P(0)$ i.e at origin $(z=0)$. Then the power at a distance z is given by $P(z)=P(0)e^{-\alpha_p z}$ where α_p is fiber attenuation constant (per km). $\alpha_p = \frac{1}{z}\ln\left[\frac{P(0)}{P(z)}\right] \Rightarrow \alpha_{dB/km} = 10\frac{1}{z}\ln\left[\frac{P(0)}{P(z)}\right]$

$$10 = 10\frac{1}{10}\log\frac{P(0)}{P(d)} \Rightarrow \frac{P(0)}{P(d)} = 10^{10} \Rightarrow \frac{E(0)}{E(d)} = \sqrt{10^{10}} = 10^5$$

Q54. A perfectly conducting fluid of permittivity ε and permeability μ flows with a uniform velocity $\vec{v}$ in the presence of time dependent electric and magnetic fields $\vec{E}$ and $\vec{B}$, respectively, if there is a finite current density in the fluid, then

(a) $\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) = \frac{\partial\vec{B}}{\partial t}$ (b) $\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) = -\frac{\partial\vec{B}}{\partial t}$

(c) $\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) = \sqrt{\varepsilon\mu}\,\frac{\partial\vec{E}}{\partial t}$ (d) $\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) = -\sqrt{\varepsilon\mu}\,\frac{\partial\vec{E}}{\partial t}$

Ans. 54: (a)

Solution: The generalised Ohm's law for conducting fluids is given by $\vec{J} = \sigma\vec{E} + \sigma\vec{v}\times\vec{B}$

If, there is no net current, $\vec{J}=0$. Thus, the above equation becomes,

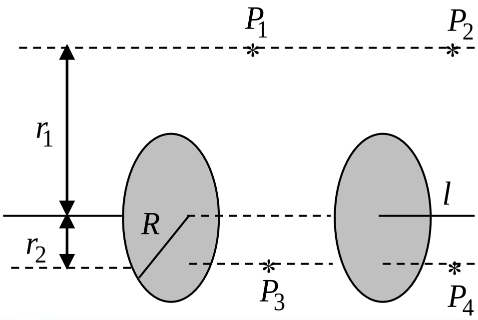
$\vec{E}+\vec{v}\times\vec{B}=0$ σ being common, cancel's out.

Taking curl of the above equation, we get $\vec{\nabla}\times\left(\vec{E}+\vec{v}\times\vec{B}\right) = \vec{\nabla}\times\vec{E} + \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) = 0$

Using $\vec{E}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ in the above equation, we get $-\frac{\partial\vec{B}}{\partial t} + \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) = 0 \Rightarrow \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) = \frac{\partial\vec{B}}{\partial t}$

Thus, (a) is the correct answer.

Q63. The figure below shows an ideal capacitor consisting of two parallel circular plates of radius R. Points P_1 and P_2 are at a transverse distance, $r_1 > R$ from the line joining the centers of the plates, while points P_3 and P_4 are at a transverse distance $r_2 < R$.



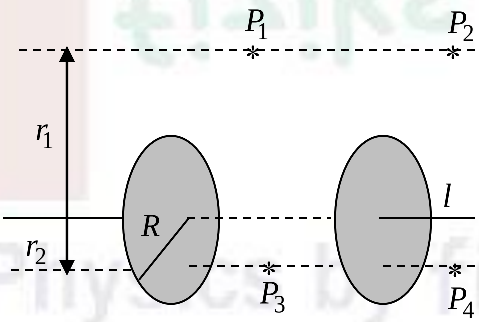
It $B(x)$ denotes the magnitude of the magnetic fields at these points, which of the following holds while the capacitor is charging?

(a) $B(P_1) < B(P_2)$ and $B(P_3) < B(P_4)$ (b) $B(P_1) > B(P_2)$ and $B(P_3) > B(P_4)$

(c) $B(P_1) = B(P_2)$ and $B(P_3) < B(P_4)$ (d) $B(P_1) = B(P_2)$ and $B(P_3) > B(P_4)$

Ans. 63: (c)

Solution:



Magnetic field at P_2 and P_4 can be simply written using Ampere's law (as these points are outside the capacitor, therefore magnetic field only depends upon the magnitude of free current which is just I).

Thus, $B_2(r) = \dfrac{\mu_0 I}{2\pi r_1}$ and $B_4(r) = \dfrac{\mu_0 I}{2\pi r_2}$

At P_1 and P_3, magnetic field depends upon displacement current.

Field at P_1: -

Using $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0\varepsilon_0 \int_S \dfrac{d\vec{E}}{dt}.d\vec{a}$, $E = \dfrac{\sigma}{\varepsilon_0}, A = \pi R^2$

The conduction current is zero. Further, note that the displacement current does not flow outside the plates, therefore $r = R$ on R.H.S and $r = r_1$ on L.H.S.

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Thus, we get $B_1 \times 2\pi r_1 = \mu_0 \varepsilon_0 \dfrac{d}{dt}\left(\dfrac{\sigma}{\varepsilon_0}\pi R^2\right) = \dfrac{\mu_0 \varepsilon_0}{\varepsilon_0}\dfrac{d}{dt}(q) = \mu_0 I \;,\left(\pi R^2 \sigma = q\right) \Rightarrow B_1 = \dfrac{\mu_0 I}{2\pi r_1}$

Field at P$_3$: -

Using $\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \int_S \dfrac{d\vec{E}}{dt}.d\vec{a}$, $E = \dfrac{\sigma}{\varepsilon_0}, A = \pi r_2^2$

Note that displacement current flowing through only $r = r_2$ counts on R.H.S. Therefore $r = r_2$ on R.H.S as well as on L.H.S.

Thus, we get

$$B_3 \times 2\pi r_2 = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\frac{\sigma}{\varepsilon_0}\pi r_2^2\right) = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\pi R^2 \varepsilon_0}\pi r_2^2\right) = \mu_0 \varepsilon_0 \frac{d}{dt}(q)\frac{\pi r_2^2}{\pi R^2 \varepsilon_0}, \left(\sigma = \frac{q}{\pi R^2}, I = \frac{dq}{dt}\right)$$

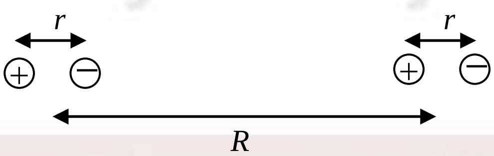
$$\Rightarrow B_3 \times 2\pi r_2 = \mu_0 I \frac{r_2^2}{R^2} \Rightarrow B_3 = \frac{\mu_0 I r_2}{2\pi R^2}$$

Comparing, $B_1 = B_2$

and $\dfrac{B_3}{B_4} = \dfrac{\mu_0 I r_2}{2\pi R^2}\dfrac{2\pi r_2}{\mu_0 I} = \dfrac{r_2^2}{R^2} < 1 (\because r_2 < R) \qquad \Rightarrow B_3 < B_4$

Thus, (c) is the correct answer.

Q66. A linear diatomic molecule consists of two identical small electric dipoles with an equilibrium separation R, which is assumed to be a constant. Each dipole has charges $\pm q$ of mass m separated by r when the molecule is at equilibrium. Each dipole can execute simple harmonic motion of angular frequency ω



Recall that the interaction potential between two dipoles of moments $\vec{p}_1$ and $\vec{p}_2$, separated by $\vec{R}_{12} = R_{12}\hat{n}$ is $\left(\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{n})(\vec{p}_2 \cdot \hat{n})/(4\pi \in_0 R_{12}^3\right)$.

Assume that $R \gg r$ and let $\Omega^2 = \dfrac{q^2}{4\pi \in_0 mR^3}$. The angular frequencies of small oscillations of the diatomic molecule are

(a) $\sqrt{\omega^2 + \Omega^2}$ and $\sqrt{\omega^2 - \Omega^2}$

(b) $\sqrt{\omega^2 + 3\Omega^2}$ and $\sqrt{\omega^2 - 3\Omega^2}$

(c) $\sqrt{\omega^2 + 4\Omega^2}$ and $\sqrt{\omega^2 - 4\Omega^2}$

(d) $\sqrt{\omega^2 + 2\Omega^2}$ and $\sqrt{\omega^2 - 2\Omega^2}$

Ans. 66: (c)

Solution:

We need to remember that for two coupled oscillators (two equal masses attached by a spring of force constant κ and attached to the walls from two sides with a spring of force constant k), the difference of squares of allowed frequency of oscillations is given by

$$\omega_2^2 - \omega_1^2 = 2\frac{\kappa}{m}, \quad \omega_1 = \sqrt{\frac{k}{m}} \text{.............(1)}$$

The situation here is identical. The interaction energy of two dipoles which are parallel is given by (given in the statement of the problem and taking the angle between the parallel dipoles to be zero degree)

$$= \frac{-2p^2}{4\pi\varepsilon_0 R^3}, p_1 = p_2 = p = qr \qquad \Rightarrow U = \frac{-2q^2r^2}{4\pi\varepsilon_0 R^3}$$

$$F = -\frac{\partial U}{\partial r} = \frac{-4q^2 r}{4\pi\varepsilon_0 R^3} = -\kappa r \qquad \left(\kappa = \frac{4q^2}{4\pi\varepsilon_0 R^3}\right)$$

Therefore, substituting the value of force constant obtained above in the (1), we get

$$\omega_2^2 - \omega_1^2 = \frac{2}{m}\frac{4q^2}{4\pi\varepsilon_0 R^3} = \frac{8q^2}{4\pi\varepsilon_0 m R^3} = 8\Omega^2, \qquad \left(\Omega^2 = \frac{q^2}{4\pi\varepsilon_0 m R^3}\right)$$

The value of Ω^2 is given in the statement of the problem. This is the difference expected in the two frequencies. If we look for this difference of frequencies in the given options, only (c) satisfies this criterion. Therefore, it is the correct option.

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Part B

Q21. Which of the following two physical quantities cannot be measured simultaneously with arbitrary accuracy for the motion of a quantum particle in three dimensions?

(a) square of the radial position and z -component of angular momentum (r^2 and L_z)

(b) x - components of linear and angular momenta (p_x and L_x)

(c) y -component of position and z -component of angular momentum (y and L_x)

(d) squares of the magnitudes of the linear and angular momenta (p^2 and L^2)

Ans. 21: (c)

Solution: The two physical quantities cannot be measured simultaneously with arbitrary accuracy in quantum mechanics whose commutator is not zero.

(a) $\left[r^2, L_z\right] = \left[x^2+y^2+z^2, L_z\right] = \left[x^2, L_z\right] + \left[y^2, L_z\right] + \left[z^2, L_z\right]$

$$= x[x, L_z] + [x, L_z]x + y[y, L_z] + [y, L_z]y = x(-i\hbar y) + (-i\hbar yx) + y(i\hbar x) + (i\hbar x)y = 0$$

where, we have used $\left[x, L_y\right] = -i\hbar y : [y, L_z] = i\hbar x \; ; [z, L_z] = 0$

(b) $[p_x, L_x] = \left[p_x, yp_z, zp_y\right] = [p_x, yp_z] - \left[p_x - zp_y\right]$

$$= y[p_x, p_z] + [p_x, y]p_z + z[p_x, p_z] - [p_x, z]p_y$$

$[p_x, L_x] = 0$

where, we have used $[p_x, p_z] = [p_x, y] = [p_x, z] = 0$

(c) $\left[p^2, L^2\right] = \left[p^2, r^2p^2 - (\vec{r}\cdot\vec{p})^2 + i\hbar(\vec{r}\cdot\vec{p})\right]$

$$= \left[p^2, r^2p^2\right] - \left[p^2, (\vec{r}\cdot\vec{p})^2\right] + i\hbar\left[p^2, (\vec{r}\cdot\vec{p})\right] = 0$$

where, we have used $[p, r] = 0$

(d) $[y, L_z] = \left[y, xp_y - yp_x\right] = \left[y, xp_y\right] - [y, yp_x] = x\left[y, p_y\right] + [y, x]p_y - y[y, p_x] - [y, y]p_x$

$$= x\left[y, p_y\right] + 0 + 0 + 0 = i\hbar x$$

where we have used $\left[y, p_y\right] = i\hbar, \; [y, x] = [y, p_x] = [y, y] = 0$

Q31. A particle of mass m is in a one dimensional infinite potential well of length L, extending from $x = 0$ to $x = L$. When it is in the energy Eigen-state labelled by $n, (n = 1, 2, 3, ..)$ the probability of finding in the interval $0 \le x \le L/8$ is 1/8. The minimum value of n for which this is possible is

(a) 4 (b) 2 (c) 6 (d) 8

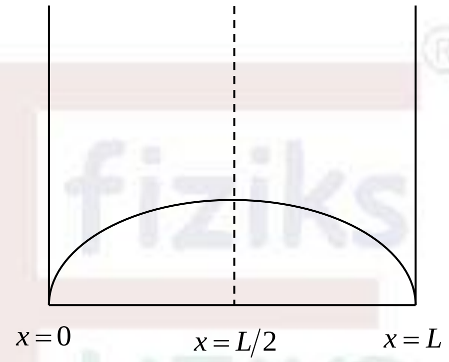
Ans. 31: (a)

Solution: This problem is solved using the wavefunction.

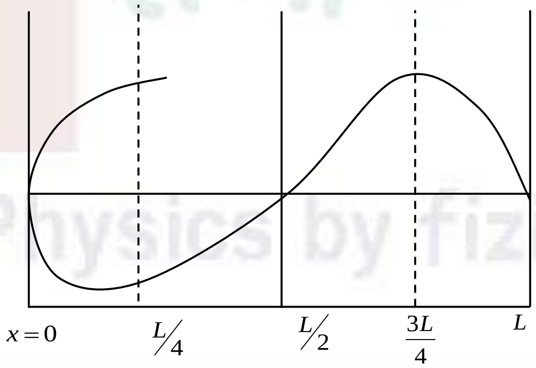
(a) The plot for $\psi_1(x)$ between $0<x<L$ is

The probability of finding the particle in region $0<x<L/2$ and $\frac{L}{2}<x<L$ is

$$P\left(0<x<L/2\right)=P\left(\frac{L}{2}<x<L\right)=\frac{1}{2}$$



(b) The plot for $\psi_2(x)$ in between $0<x<L$



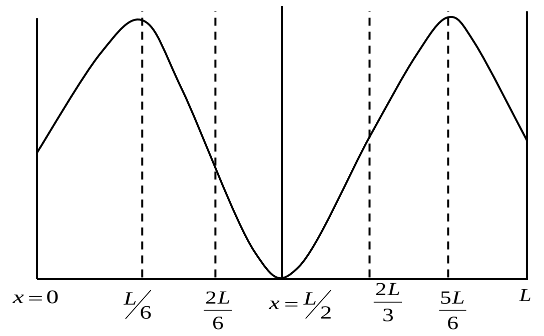
The probability of finding the particle in region.

$0<x<L/4$;

$\frac{L}{4}<x<\frac{L}{2};\ \frac{L}{2}<x<\frac{3L}{4};\ \frac{3L}{4}<x<L$ is

$$P\left(0<x<\frac{L}{4}\right)=P\left(\frac{L}{4}<x<\frac{L}{2}\right)=P\left(\frac{L}{2}<x<\frac{3L}{4}\right)=P\left(\frac{3L}{4}<x<L\right)=\frac{1}{4}$$

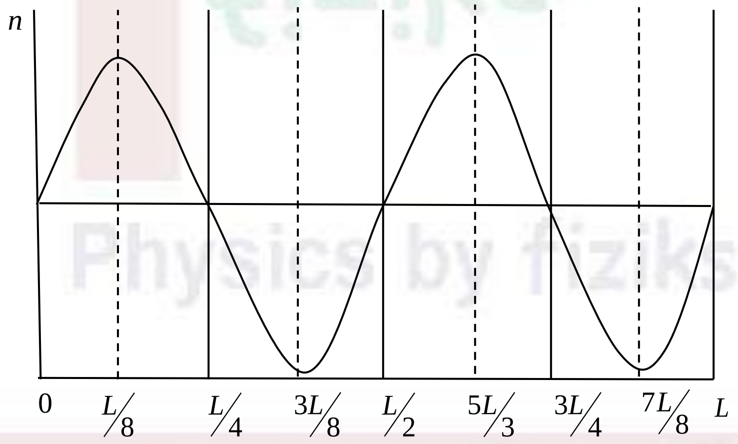
(c) The plot for $\psi_3(x)$ in region $0<x<L$ is



The plot is divided in 6 equal region of $0<x<L/6$; $\frac{L}{6}<x<2L/6$ $\frac{2L}{6}<x<\frac{L}{2}$; $\frac{L}{2}<x<\frac{2L}{3}$; $\frac{2L}{3}<x<\frac{5L}{6}$; $\frac{5L}{6}<x<L$.

The probability of finding the particle in each of region is $1/6$.

(d) The plot for $\psi_4(x)$ in region $0<x<L$ is



The wave function is divided in 8 equal region of $0<x<\frac{L}{8}, \frac{L}{8}<x<\frac{L}{4}$, $\frac{L}{4}<x<\frac{3L}{8}$, $\frac{3L}{8}<x<\frac{L}{2}$, $\frac{L}{2}<x<\frac{5L}{8}$, $\frac{5L}{8}<x<\frac{3L}{4}$, $\frac{3L}{4}<x<\frac{7L}{8}$, $\frac{7L}{8}<x<L$.

The probability of finding the particle in each of these region is $1/8$.

Thus, the value of $n=4$, such that the probability of finding the particle in region $P\left(0<x<L/8\right)=1/8$.

Q38. A two-state system evolves under the action of the Hamiltonian $H = E_0|A\rangle\langle A| + (E_0+\Delta)|B\rangle\langle B|$, where $|A\rangle$ and $|B\rangle$ are its two orthonormal states. These states transform to one another under parity, i.e. $P|A\rangle = |B\rangle$ and $P|B\rangle = |A\rangle$. If at time $t=0$ the system is in a state of definite parity $P=1$, the earliest time t at which the probability of finding the system in a state of parity $P=-1$ is one is

(a) $\frac{\pi\hbar}{2\Delta}$ (b) $\frac{\pi\hbar}{\Delta}$ (c) $\frac{3\pi\hbar}{2\Delta}$ (d) $\frac{2\pi\hbar}{\Delta}$

Ans. 38: (b)

Solution.: The Hamiltonian for the two state system is given by $H = \varepsilon_0|A\rangle\langle A| + (\varepsilon_0 + D)|B\rangle\langle B|$

In matrix, $H = \begin{pmatrix} E_0 & 0 \\ 0 & \varepsilon_0 + D \end{pmatrix}$

The energy eigenvalue for the system is given by $|H - \lambda I| = \begin{vmatrix} \varepsilon_0 - \lambda & 0 \\ 0 & (\varepsilon_0 + 0) - \lambda \end{vmatrix} = 0$

or $\lambda = \varepsilon_0,\ \varepsilon_0 + 0$

The eigenfunction of the system is given by $|\phi(t)\rangle = |A\rangle e^{-\frac{iE_0}{\hbar}t} + |B\rangle e^{\frac{-i(\varepsilon_0+0)}{\hbar}t}$

According to question, we have $\pi|\phi(t)\rangle = -|\phi(t)\rangle$

$$\pi|A\rangle e^{-i\varepsilon_0 t/\hbar} + \pi|B\rangle e^{\frac{-i(\varepsilon_0+D)}{\hbar}t} = -|A\rangle e^{-i\varepsilon_0 t/\hbar} - |B\rangle e^{\frac{-(\varepsilon_0+D)}{\hbar}t}$$

$$|B\rangle e^{\frac{-i\varepsilon_0 t}{\hbar}} + |A\rangle e^{\frac{-i(\varepsilon_0+D)t}{\hbar}} = -|A\rangle e^{-i\varepsilon_0 t/\hbar} - |B\rangle e^{\frac{-i(E_0+D)}{\hbar}t}$$

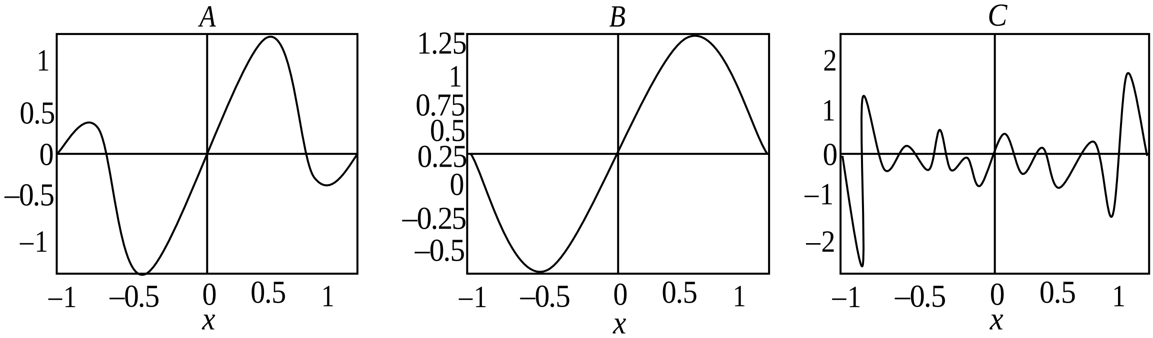
Comparing coefficient of state $|A\rangle$ and $|B\rangle$, we get

For A: $-e^{-i\varepsilon_0 t/\hbar} = e^{\frac{-i(\varepsilon_0+D)t}{\hbar}}$; For B: $-e^{\frac{-i(E_0+D)t}{\hbar}} = e^{-i\varepsilon_0 t/\hbar}$

Since both these conditions are same, we $-e^{-i\varepsilon_0 t/\hbar} = e^{-i(\varepsilon_0+0)t/\hbar}$ or $e^{-i(\varepsilon_0+0)t/\hbar}e^{-i\varepsilon_0 t/\hbar} - 1$

$e^{\frac{i\Delta t}{\hbar}} = e^{i\pi} \Rightarrow i\frac{\Delta t}{\hbar} = i\pi$ or $t = \frac{\hbar\pi}{\Delta}$ Thus the correct option is (b)

Q42. The figures below depict three different wave functions of a particle confined to a one dimensional box $-1 \le x \le 1$



The wave functions that correspond to the maximum expectation values $|\langle x\rangle|$ (absolute value of the mean position) and $\langle x^2\rangle$, respectively, are

(a) B and C (b) B and A (c) C and B (d) A and B

Ans. 42: (a)

Solution: This problem is solved using properties:

(1) For a box of length $-a < x < d$, $\langle x\rangle$ is always zero.

(2) For a $60x$ of length $-a < x < d$, $|\langle x\rangle|$ is always non zero.

(3) The wavefunction is of the form $\psi(x) = A\left(a^2 - x^2\right) \qquad x = \pm a$

The normalised wave function is given by $\psi(x) = \sqrt{\frac{15}{16a^3}}\left(a^2 - x^2\right)$

The expectation value of $\langle x^2\rangle$ is $\langle x^2\rangle = \frac{15}{16a^5}\int_{-a}^{d} x^2\left(a^2 - x^2\right)dx = \frac{16a^7}{105}$

Thus, at $a = \pm 1$ curve would take maximum and minimum values.

For $|\langle x\rangle|$ the curve given in the option (b) is non-zero.

For $\langle x^2\rangle$, the curve takes maximum and minimum value at $a = \pm 1$ in the curve shown in option (c).

Q43. The Hamiltonian of a particle of mass m in one-dimension is $H = \frac{1}{2m}p^2 + \lambda|x|^3$, where $\lambda > 0$ is a constant. If E_1 and E_2 respectively, denote the ground state energies of the particle for $\lambda = 1$ and $\lambda = 2$ (in appropriate units) the ratio E_2 / E_1 is best approximated by

(a) 1.260 (b) 1.414 (c) 1.516 (d) 1.320

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Ans. 43: (d)

Solution: Consider the potential of the particle of form $V(x)=\lambda|x|^n$

The ground state energy of the particle using with approximation depends on λ or $\varepsilon_n \alpha \lambda^{\frac{2}{\lambda+2}}$

So the ration $\varepsilon_2 | \varepsilon_1$ is given by $\frac{\varepsilon_2}{\varepsilon_1}=\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{2}{m+2}}=\left(\frac{2}{1}\right)^{\frac{2}{3+2}}=2^{2/5}$ or $\frac{\varepsilon_2}{\varepsilon_1}=1.319\approx 1.32$.

Part C

Q49. The energies of a two-state quantum system are E_0 and $E_0+\alpha\hbar$, (where $\alpha>0$ is a constant) and the corresponding normalized state vectors are $|0\rangle$ and $|1\rangle$, respectively. At time $t=0$, when the system is in the state $|0\rangle$, the potential is altered by a time independent term V such that $\langle 1|V|0\rangle=\hbar\alpha/10$. The transition probability to the state $|1\rangle$ at times $t \ll 1/\alpha$, is

(a) $\alpha^2t^2/25$ (b) $\alpha^2t^2/50$ (c) $\alpha^2t^2/100$ (d) $\alpha^2t^2/200$

Ans. 49: (c)

Solution: The transmission probability to the state $|1\rangle$ at time t is

$$P_{0\to1}=\frac{1}{\hbar^2}\left|\langle 1|\forall|0\rangle\right|^2 \Big/ \int_0^t e^{i\left(\frac{\varepsilon_1-\varepsilon_0}{\hbar}\right)t}\,dt$$

$$=\frac{1}{\hbar^2}\left(\frac{\hbar\alpha}{10}\right)^2\left|\int_0^t e^{i}\,\frac{\alpha\hbar}{\hbar}t\,dt\right|^2=\frac{\alpha^2}{100}\left|\int_0^t e^{i\alpha t}dt\right|^2=\frac{\alpha^2}{100}\left|\int_0^t dt\right|^2=\frac{\alpha^2t^2}{100}$$

where, we have used $e^{i\alpha t}\approx 1$ as $\alpha t \ll 1$

Q61. A particle of mass m in one dimension is in the ground state of a simple harmonic oscillator described by a Hamiltonian $H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2$ in the standard notation. An impulsive force at time to $t=0$ suddenly imparts a momentum $p_0=\sqrt{\hbar m\omega}$ to it. The probability that the particle remains in the original ground state is

(a) e^{-2} (b) $e^{-3/2}$ (c) e^{-1} (d) $e^{-1/2}$

Ans. 61: (d)

Solution: The new state of the system is $\psi_{P_0}(x)=e^{-ip_0x/\hbar}\psi_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}e^{-ip_0x\hbar}e^{-m\omega x^2/2\hbar}$

In an expansion in the complete set of harmonic oscillator eigenfunction.

$$\psi_{P_0}(x)=\sum_{n=0}^{\infty}C_n\psi_n(x)$$

the coefficient $C_n=\int_{-\infty}^{\infty}dx\,\psi_n^*(x)\psi_{P_0}(x)$

are the probability amplitudes for the system in the state ψ_n. Thus

$$P_0=\left|\int\psi_0(x)\psi_{P_0}(x)dx\right|^2=\left|\int\psi_0^2(x)e^{-|p_0x|\hbar}dx\right|^2$$

Calculating the Guarian integral $\int_{-\infty}^{\infty}dx\,e^{\left(-\frac{i}{\hbar}xp_0-\frac{m\omega}{\hbar}x^2\right)}=\sqrt{\frac{\pi}{t}}e^{-\frac{(P_0/\hbar)^2}{4(m\omega/\hbar)}}=\sqrt{\frac{\pi\hbar}{m\omega}}e^{-\frac{p_0^2}{4m\omega\hbar}}$

Substituting value in expression of probability.

$$P_0\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{\infty}e^{\left(-\frac{i}{\hbar}p_0x-\frac{m\omega}{\hbar}x^2\right)}dx\Bigg|^2=\left|\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}\left(\frac{\pi\hbar}{m\omega}\right)^{1/2}e^{-\frac{p_0^2}{4m\omega\hbar}}\right|^2=e^{-\frac{p_0^2}{2m\omega\hbar}}$$

We get $P_0=e^{-\frac{p_0^2}{2m\omega\hbar}}=e^{-\frac{\left(\sqrt{m\omega\hbar}\right)^2}{2m\omega\hbar}}=e^{-1/2}$

where, we have used $P_0=\sqrt{m\omega\hbar};\ \int_{-\infty}^{\infty}e^{-i\alpha x-\beta x^2}dx=\sqrt{\frac{\pi}{\beta}}e^{-\frac{\alpha^2}{4\beta}}$

Q64. The $|3,0,0\rangle$ state in the standard notation $|n,l,m\rangle$ of the H-atom in the non-relativistic theory decays to the state $|1,0,0\rangle$ via two dipole transition. The transition route and the corresponding probability are

(a) $|3,0,0\rangle\to|2,1,-1\rangle\to|1,0,0\rangle$ and $1/4$ (b) $|3,0,0\rangle\to|2,1,1\rangle\to|1,0,0\rangle$ and $1/4$

(c) $|3,0,0\rangle\to|2,1,0\rangle\to|1,0,0\rangle$ and $1/3$ (d) $|3,0,0\rangle\to|2,1,0\rangle\to|1,0,0\rangle$ and $2/3$

Ans. 64: (c)

Solution: For, dipole transition,

$\Delta l=\pm1$ *and* $\Delta m=0,\pm1$

For all options, n = 2, so l = 0, 1

For l = 0, m = 0 and for l = 1, m = -1, 0, 1

The transitions $|3,0,0\rangle\longrightarrow|1,0,0\rangle$ via $|2,1,m\rangle$ for m = -1, 0, 1 are all valid according to the dipole transition rule. Thus, there are three different states through which the $|3,0,0\rangle$ state can decay to $|1,0,0\rangle$ each with equal probability. Hence each transition has a probability of 1/3. So, option (c) with probability 1/3 is correct.

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Q74. In an elastic scattering process at an energy E, the phase shifts satisfy $\delta_0 \approx 30^\circ, \delta_1 \approx 10^\circ$, while the other phase shifts are zero. The polar angle at which the differential cross section peaks is closest to

(a) 20° (b) 10° (c) 0° (d) 30°

Ans. 74: (c)

Solution: In the partial wave expansion, the differential scattering cross section is given by

$$\frac{d\theta}{d(\cos\theta)} = \left|\sum_\ell (2\ell+1) e^{i\delta e} \sin \delta e \psi_\ell (\cos\theta)\right|^2$$

where θ is the scattering angle. Taking 'Cross section for $\ell = 0$ and $\ell = 1$, we have,

$$\frac{d\theta}{d(\cos\theta)} = \left|e^{i\delta\theta} \sin \delta\theta + 3e^{i\delta_1} \sin \delta_1 \cos\theta\right| = 0$$

Since the differential cross section peaks is do not,

$$\frac{d\theta}{d(\cos\theta)} = \left|e^{i\delta\theta} \sin \delta\theta + 3e^{i\delta_1} \sin \delta_1 \cos\theta\right| = 0 \quad \text{or } \cos\theta = \frac{-e^{i\delta\theta} \sin \delta\theta}{3e^{i\delta_1} \sin \delta_1}$$

Simplifying above expression

$$\cos\theta = \frac{-1}{3} \frac{2\cos\delta\theta \sin\delta\theta}{2\cos\delta_1 \sin\delta_1} = -\frac{1}{3}\frac{\sin 2\delta\theta}{\sin 2\delta_1} \Rightarrow \cos\theta = -\frac{1}{3}\frac{\sin 2\times 30}{\sin 2\times 10} = -\frac{1}{3}\frac{\sin 60}{\sin 20}$$

$$\Rightarrow \cos\theta = -\frac{1}{3} \times \frac{0.8660}{0.342} = -0.844 \quad \Rightarrow \cos\theta = 0.844 \Rightarrow \theta = 32^\circ.4$$

Thus, the closest angle would be 30°.

Q75. The unnormalized wave function of a particle in one dimension in an infinite square well with walls at $x=0$ and $x=a$, is $\psi(x) = x(a-x)$. If $\psi(x)$ is expanded as a linear combination of the energy eigenfunctions, $\int_0^a |\psi(x)|^2 dx$ is proportional to the infinite series

(You may use $\int_0^a t \sin t\, dt = -a\cos a + \sin a$ and $\int_0^a t^2 \sin t\, dt = -2 - (a^2 - 2)\cos a + 2a \sin a$

(a) $\sum_{n=1}^{\infty} (2n-1)^{-6}$ (b) $\sum_{n=1}^{\infty} (2n-1)^{-4}$ (c) $\sum_{n=1}^{\infty} (2n-1)^{-2}$ (d) $\sum_{n=1}^{\infty} (2n-1)^{-8}$

Ans. 75: (a)

Solution: We have, $\psi(x_1, t=0) = \alpha(a-x)$

The normalization constant is determined at follows.

$$\int |\psi(x)|^2 dx = A^2 \int_0^a x^2 (a-x)^2 dx = A^2 \frac{a^5}{30} = 1 \quad \text{or } A = \sqrt{\frac{a^5}{30}}$$

Thus, the normalised wave function is given by $\psi\left(x_1, t=0\right)=x(a-x)\cdot\sqrt{\frac{30}{a^5}}$

We expand $\psi(x, t=0)$ in terms of energy eigen state $\psi(x, 0)=\sum_{n=1}^{\infty} C_n \psi_n(x)$.

Multiply the above equation by $\psi_n^*(x)$ and integrate to determine coefficient C_n,

$$C_n=\int_0^L \psi_n^*(x) \psi(x, 0)\, dx=\left(\frac{30}{a^5}\right)^{1/2}\left(\frac{2}{a}\right) \int_0^a x(a-x) \sin \frac{n \pi x}{d} dx$$

Let change of variable $y=\pi x / L$; $C_n=\frac{2 \sqrt{15}}{\pi^2} \int_0^\pi y\left(1-\frac{y}{\pi}\right) \sin n y\, d y$

Employing integral $\int_0^\pi y \sin n y=-\frac{\pi}{n}(-1)^n$, $\int_0^\pi y^2 \sin n y\, d y=-\frac{\pi^2}{n}(-1)^n+\frac{2}{n^3}\left[(-1)^n-1\right]$

We get $C_n=\frac{4 \sqrt{15}}{\pi^3 n^3}\left[1-(-1)^n\right]$. Probability P_n is given by $P_n=\left|a_n\right|^2=\frac{240}{\pi^6 n^6}\left[1-(-1)^n\right]^2$

One can see that P_n is proportional to n^{-6}, this is assessable in option (1). Hence the correct series would by $\sum_{n=1}^{\infty}(2 n-1)^{-6}$.

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Part-B

Q31. If the average energy $\langle E\rangle_T$ of a quantum harmonic oscillator at a temperature T is such that $\langle E\rangle_T = 2\langle E\rangle_{T\to 0}$, then T satisfies

(a) $\coth\left(\frac{\hbar\omega}{k_B T}\right) = 2$

(b) $\coth\left(\frac{\hbar\omega}{2k_B T}\right) = 2$

(c) $\coth\left(\frac{\hbar\omega}{k_B T}\right) = 4$

(d) $\coth\left(\frac{\hbar\omega}{2k_B T}\right) = 4$

Ans.: (b)

Solution:

For Quantum Harmonic Oscillator $\varepsilon_n = \left(n+\frac{1}{2}\right)\hbar\omega$, $n = 0,1,2,3,\ldots$

$$Z = \sum_{n=0}^{\infty} e^{-\beta\varepsilon_n} = \sum_{n=0}^{\infty} e^{-\beta\left(n+\frac{1}{2}\right)\hbar\omega} = e^{-\frac{\beta\hbar\omega}{2}} + e^{-\frac{3}{2}\beta\hbar\omega} + e^{-\frac{5}{2}\beta\hbar\omega} + \ldots = e^{-\frac{\beta\hbar\omega}{2}}\left[1+e^{-\beta\hbar\omega}+e^{-2\beta\hbar\omega}+\ldots\right]$$

$$\Rightarrow Z = \frac{e^{-\frac{\beta\hbar\omega}{2}}}{1-e^{-\beta\hbar\omega}} \qquad \Rightarrow \ln Z = \frac{-\beta\hbar\omega}{2} - \ln\left(1-e^{-\beta\hbar\omega}\right)$$

Thus $\langle E\rangle = -\frac{\partial}{\partial\beta}\ln Z = \frac{\hbar\omega}{2} + \frac{-e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}}(-\hbar\omega) \quad \Rightarrow \langle E\rangle_T = \left[\frac{1}{2}+\frac{1}{e^{\beta\hbar\omega}-1}\right]\hbar\omega$

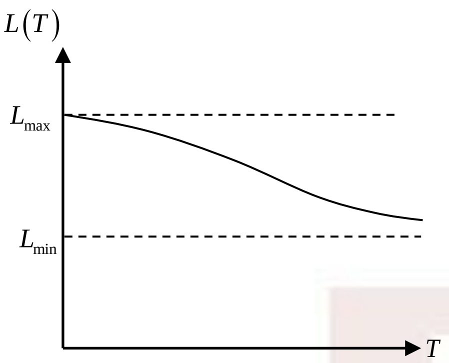
Now, as $T\to 0, \beta\to\infty, e^{\beta\hbar\omega}\to$ large; $\langle E\rangle_{T\to 0} = \frac{\hbar\omega}{2}$

$$\because \langle E\rangle_T = 2\langle E\rangle_{T\to 0} \Rightarrow \left[\frac{1}{2}+\frac{1}{e^{\beta\hbar\omega}-1}\right]\hbar\omega = \hbar\omega \Rightarrow \frac{1}{2}+\frac{1}{e^{\beta\hbar\omega}-1} = 1 \Rightarrow \frac{e^{\beta\hbar\omega}-1+2}{2\left(e^{\beta\hbar\omega}-1\right)} = 1$$

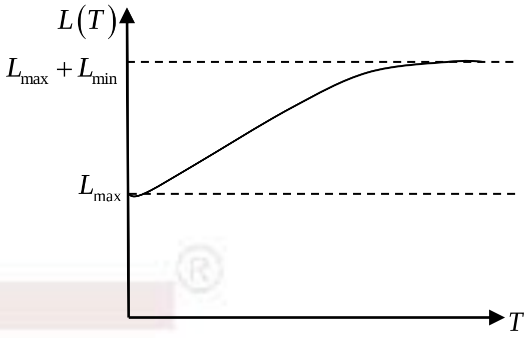
$$\Rightarrow \frac{e^{\beta\hbar\omega}+1}{e^{\beta\hbar\omega}-1} = 2 \Rightarrow \frac{e^{\frac{\beta\hbar\omega}{2}}+e^{-\frac{\beta\hbar\omega}{2}}}{e^{\frac{\beta\hbar\omega}{2}}-e^{-\frac{\beta\hbar\omega}{2}}} = 2 \Rightarrow \coth\left(\frac{\beta\hbar\omega}{2}\right) = 2 \text{ or } \coth\left(\frac{\hbar\omega}{2k_B T}\right) = 2$$

Q43. An elastic rod has a low energy state of length L_{max} and high energy state of length L_{min}. The best schematic representation of the temperature (T) dependence of the mean equilibrium length L(T) of the rod, is

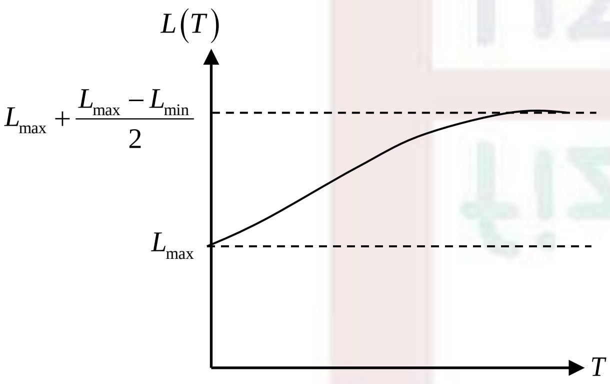
(a)



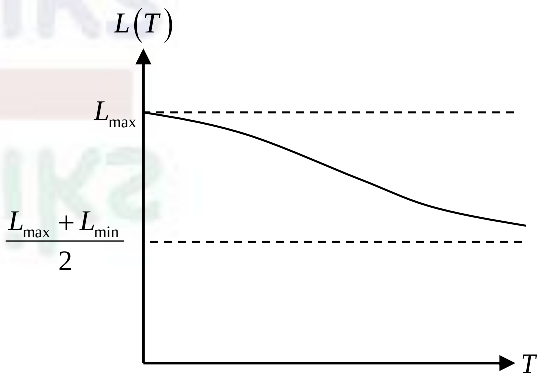
(b)



(c)



(d)



Ans.: (d)

Solution:

Let E_1 and E_2 represent the lowest and highest energy states of the elastic rod.

$E_1 = CL_{max}$ and $E_2 = CL_{min}$, where C is a constant of appropriate dimensions so that CL has dimensions of energy.

Let $P(E_1)$ and $P(E_2)$ are the probabilities of the rod to have states with energy E_1 and E_2, respectively. Then, $P(E_1) = \dfrac{e^{-\beta E_1}}{e^{-\beta E_1} + e^{-\beta E_2}}$, $P(E_2) = \dfrac{e^{-\beta E_2}}{e^{-\beta E_1} + e^{-\beta E_2}}$

$$\langle L \rangle = L_{max} P(E_1) + L_{min} P(E_2) \qquad ....(1)$$

As $T \to \infty$, $\beta \to 0$, $e^{-\beta E_1} \approx e^0 = 1$, $\quad \because \dfrac{E_1}{k_B T} \approx 0$ as E_1 is small and $T \to \infty$

$e^{-\beta E_2} \approx e^0 = 1$, $\quad \because \dfrac{E_2}{k_B T} \approx 0$, E_2 is large but $T \to \infty$

$\therefore$ In this case $P(E_1)=P(E_2)=\frac{1}{1+1}=\frac{1}{2}$ and $\langle L\rangle=\frac{L_{\max}+L_{\min}}{2}$

In the other extreme, when $T\to 0, \beta=\frac{1}{k_BT}\to\infty$

$$\langle L\rangle=\frac{e^{-\beta E_1}}{e^{-\beta E_1}+e^{-\beta E_2}}L_{\max}+\frac{e^{-\beta E_2}}{e^{-\beta E_1}+e^{-\beta E_2}}L_{\min}=\frac{e^{-\beta E_1}L_{\max}+e^{-\beta E_2}L_{\min}}{e^{-\beta E_1}+e^{-\beta E_2}}$$

$$\langle L\rangle=\frac{e^{-\beta E_1}\left[L_{\max}+e^{-\beta(E_2-E_1)}\right]}{e^{-\beta E_1}\left[1+e^{-\beta(E_2-E_1)}\right]}=\frac{L_{\max}+e^{-\beta(E_2-E_1)}}{1+e^{-\beta(E_2-E_1)}}\approx L_{\max},\qquad \beta\to\infty \text{ as } T\to 0$$

Q45. A thermally isolated container, filled with an ideal gas at temperature *T*, is divided by a partition, which is clamped initially, as shown in the figure below.

The partition does not allow the gas in the two parts to mix. It is subsequently released and allowed to move freely with negligible friction. The final pressure at equilibrium is

(a) 5*P*/3 (b) 5*P*/4

(c) 3*P*/5 (d) 4*P*/5

Ans. :(a)

Solution:

$\because$ Vessel is isolated, $\Delta Q=0$. Since both partitions are at same temperature, we have for left and right part,

$PV=n_1RT$ ....(i)

$4PV=n_2RT$ ....(ii)

Here, we considered number of moles to be different in two parts.

Adding (i) and (ii)

$5PV=(n_1+n_2)RT\Rightarrow(n_1+n_2)=\frac{5PV}{RT}$ ....(iii)

Now, after mixing, let P_f be the final pressure, then

$$P_f(3V)=(n_1+n_2)RT\quad\Rightarrow P_f=\frac{(n_1+n_2)RT}{3V}=\frac{5PV}{3V}\frac{RT}{RT}$$

$P_f=\frac{5}{3}P$, $\therefore$ (a) is correct.

Part-C

Q49. A system of N non-interacting particles in one-dimension, each of which is in a potential $V(x) = gx^6$ where $g > 0$ is a constant and x denotes the displacement of the particle from its equilibrium position. In thermal equilibrium, the heat capacity at constant volume is

(a) $\frac{7}{6}Nk_B$ (b) $\frac{4}{3}Nk_B$ (c) $\frac{3}{2}Nk_B$ (d) $\frac{2}{3}Nk_B$

Ans.: (d)

Solution: $V(x) = gx^6$; $E = \frac{p_x^2}{2m} + gx^6 \quad \Rightarrow \langle E \rangle = \left\langle \frac{p_x^2}{2m} \right\rangle + g\langle x^6 \rangle$.

For $V(x) = ax^n$, a is a constant; $\langle V \rangle = \frac{k_B T}{n}$

$$\therefore \ \langle E \rangle = \frac{k_B T}{2} + \frac{k_B T}{6} = \frac{3k_B T + k_B T}{6} = \frac{4}{6}k_B T \quad \Rightarrow \langle E \rangle = \frac{2}{3}k_B T$$

For N such non-interacting particles $U = N\langle E \rangle = \frac{2}{3}Nk_B T \ \Rightarrow C_V = \left(\frac{dU}{dT}\right)_V = \frac{2}{3}Nk_B$

Q55. The energy levels of a system, which is in equilibrium at temperature $T = 1/(k_B \beta)$, are 0, $\in$ and $2\in$. If two identical bosons occupy these energy levels, the probability of the total energy being $3\in$, is

(a) $\frac{e^{-3\beta\in}}{1+e^{-\beta\in}+e^{-2\beta\in}+e^{-3\beta\in}+e^{-4\beta\in}}$ (b) $\frac{e^{-3\beta\in}}{1+2e^{-\beta\in}+2e^{-2\beta\in}+e^{-3\beta\in}+e^{-4\beta\in}}$

(c) $\frac{e^{-3\beta\in}}{e^{-\beta\in}+2e^{-2\beta\in}+e^{-3\beta\in}+e^{-4\beta\in}}$ (d) $\frac{e^{-3\beta\in}}{1+e^{-\beta\in}+2e^{-2\beta\in}+e^{-3\beta\in}+e^{-4\beta\in}}$

Ans.: (d)

Solution:

Two Bosons can be distributed in three energy levels as below

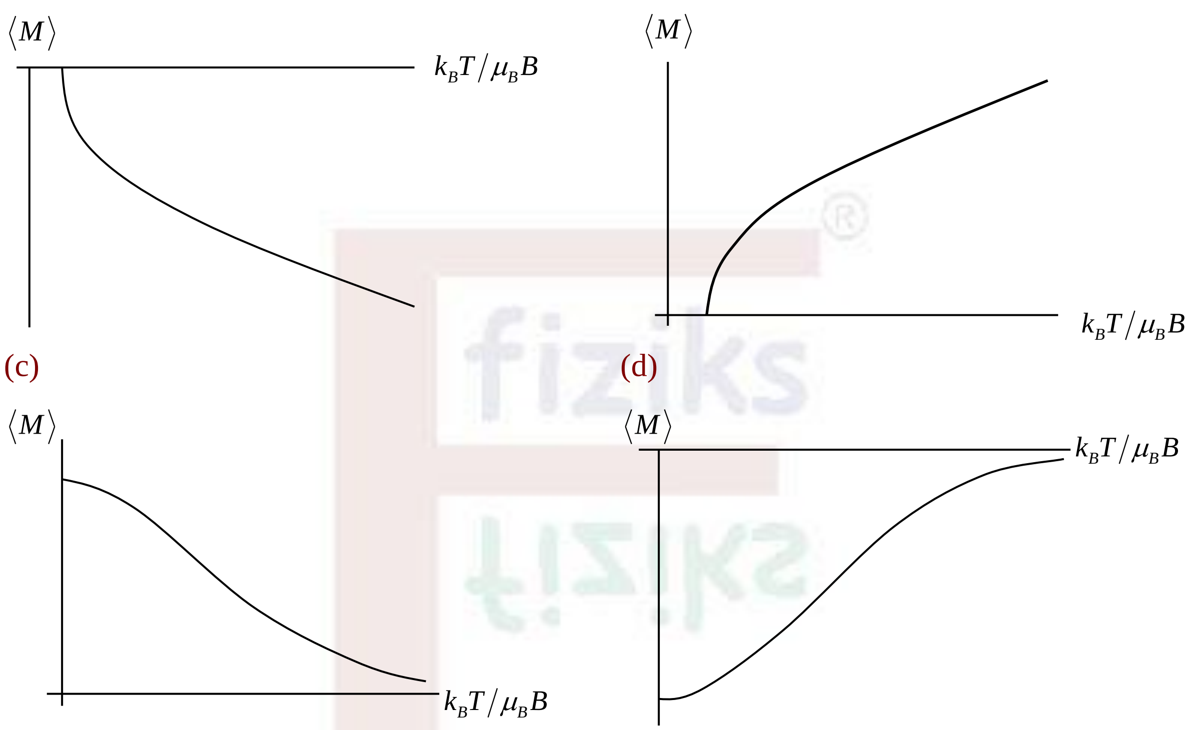
		00		0	0	2ε
	00		0	0	0	ε
00			0			0
0	2ε	4ε	ε	2ε	3ε	

There are six microstates, out of which only one has energy 3ε . Corresponding probability is

$$P(3\varepsilon) = \frac{e^{-3\beta\varepsilon}}{Z} = \frac{e^{-3\beta\varepsilon}}{1+e^{-\beta\varepsilon}+2e^{-\beta\varepsilon}+e^{-3\beta\varepsilon}+e^{-4\beta\varepsilon}}$$

Q65. A paramagnetic salt with magnetic moment per ion $\mu_{\pm} = \pm\mu_B$ (where μ_B is the Bohr magneton) is in thermal equilibrium at temperature T in a constant magnetic field B. The average magnetic moment $\langle M \rangle$, as a function of $\frac{k_B T}{\mu_B B}$, is best represented by

(a) (b) (c) (d)



Ans.: (c)

Solution: Each spin with magnetic moment $\mu = \mu_B$ has two possible spin orientations. Corresponding interaction energies in external constant field B are $+\mu_B B$ and $-\mu_B B$. Let us denote these energies as $+\varepsilon$ and $-\varepsilon$, respectively.

$\therefore$ The partition function for single spin system is $Q_1(\beta) = e^{\beta\varepsilon} + e^{-\beta\varepsilon}$ ....(1)

$$Q_N(\beta) = [Q_1]^N = \left[e^{\beta\varepsilon} + e^{-\beta\varepsilon}\right]^N \Rightarrow Q_N(\beta) = [2\cosh\beta\varepsilon]^N \quad ....(2)$$

The Helmholtz-free energy is $A = -k_B T \ln Q_N(\beta) = -Nk_B T \ln\left\{2\cosh\frac{\varepsilon}{kT}\right\}$ ....(3)

The magnetization is obtained as below:

$$\langle M \rangle = \frac{N}{\beta}\frac{\partial}{\partial B}\ln Q_1 = Nk_B T\frac{\partial}{\partial B}\ln Q_1 = k_B T\frac{\partial}{\partial B}\ln Q_1^N = \frac{\partial}{\partial B}k_B T\ln Q_1^N = -\left(\frac{\partial A}{\partial B}\right)_T$$

$$\langle M \rangle = -\left(\frac{\partial A}{\partial B}\right)_T = Nk_B T\frac{\partial}{\partial B}\left[\ln 2\cosh\frac{\mu_B B}{k_B T}\right] = Nk_B T\frac{1}{2\cosh\left(\frac{\mu_B B}{k_B T}\right)}\times 2\sinh\left(\frac{\mu_B B}{k_B T}\right)\left(\frac{\mu_B}{k_B T}\right)$$

$$\Rightarrow \langle M \rangle = N\mu_B \tanh\left(\frac{\mu_B B}{k_B T}\right) = N\mu_B \tanh\left(\frac{\varepsilon}{k_B T}\right) \qquad \text{....(4)}$$

Let us approximate $\tanh\left(\frac{\varepsilon}{k_B T}\right)$ as below

(i) High T and low B so that $\beta\varepsilon \ll 1$

$$\tanh(\beta E) \approx \beta\varepsilon - \frac{1}{3}(\beta\varepsilon)^3 + \dots , \quad \because \tanh x \approx x - \frac{x^3}{3} \dots,\ x \ll 1 \qquad \text{....(5)}$$

$M = N\mu_B \tanh(\beta\varepsilon) \approx N\mu_B (\beta\varepsilon) \to 0$ as $\beta\varepsilon \ll 1$ i.e. a state of complete randomization

(ii) Low T and high B, i.e. $\beta\varepsilon \gg 1$; $\quad \tanh(\beta\varepsilon) = \frac{e^{\beta\varepsilon} - e^{-\beta\varepsilon}}{e^{\beta\varepsilon} + e^{-\beta\varepsilon}} \approx \frac{e^{\beta\varepsilon}}{e^{\beta\varepsilon}} \approx 1$ ....(6)

$M \approx N\mu_B$, $\because \tanh(\beta\varepsilon) \approx 1$ i.e. system is completely magnetized.

$\therefore$ (c) is correct.

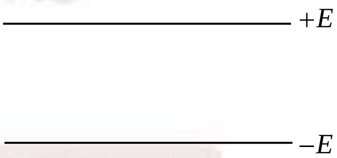
Q72. The energies of a two-level system are $\pm E$. Consider an ensemble of such non-interacting systems at a temperature *T*. At low temperatures, the leading term in the specific heat depends on *T* as

(a) $\frac{1}{T^2} e^{-E/k_B T}$ (b) $\frac{1}{T^2} e^{-2E/k_B T}$

(c) $T^2 e^{-E/k_B T}$ (d) $T^2 e^{-2E/k_B T}$

Ans.: (b)

Solution: The partition function is $Z = e^{\beta E} + e^{-\beta E} = 2\cosh(\beta E)$



$$\langle E \rangle = -\frac{\partial(\ln Z)}{\partial \beta} = -\frac{\partial}{\partial \beta} \ln\cosh(\beta E) = -\frac{1}{\cosh(\beta E)} \sinh(\beta E) \times E$$

$$\langle E \rangle = -E\tanh(\beta E)$$

$$C_V = \frac{\partial \langle E \rangle}{dT} = \frac{-d}{dT}\left[E\tanh(\beta E)\right] = -E\,\mathrm{sec}h^2(\beta E)\frac{d}{dT}\left(\frac{E}{k_B T}\right) = -E\,\mathrm{sec}h^2(\beta E) \times \left(-\frac{E}{k_B T^2}\right)$$

$$C_V = \frac{E^2}{k_B T^2}\mathrm{sec}h^2(\beta E)$$

Now, $\mathrm{sec}h^2(\beta E) = \frac{1}{\cosh^2(\beta E)} = \frac{4}{\left(e^{\beta E} + e^{-\beta E}\right)^2}$

when $T \to 0, \beta \to$ high and $e^{-\beta E}$ is low, therefore $e^{\beta E} + e^{-\beta E} \approx e^{\beta E}$, $\mathrm{sec}h^2(\beta E) \approx 4e^{-2\beta E}$

$$\therefore\ C_V \propto \frac{1}{T^2} e^{-2\beta E}$$

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Solution-Electronics and Experimental Techniques

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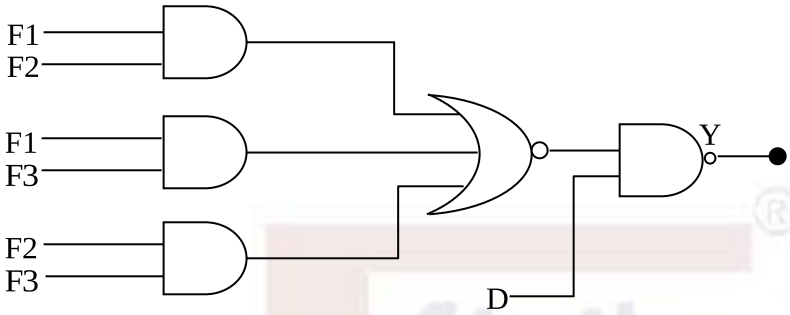
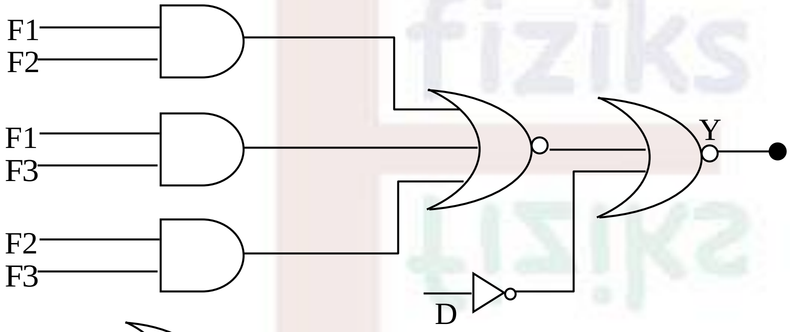
Part B

Q27. The door of an X -ray machine room is fitted with a sensor D (0 is open and 1 is closed). It is also equipped with three fire sensors F_1, F_2 and F_3 (each is 0 when disabled and 1 when enabled). The X -ray machine can operate only if the door is closed and at least 2 fire sensors are enabled. The logic circuit to ensure that the machine can be operated is

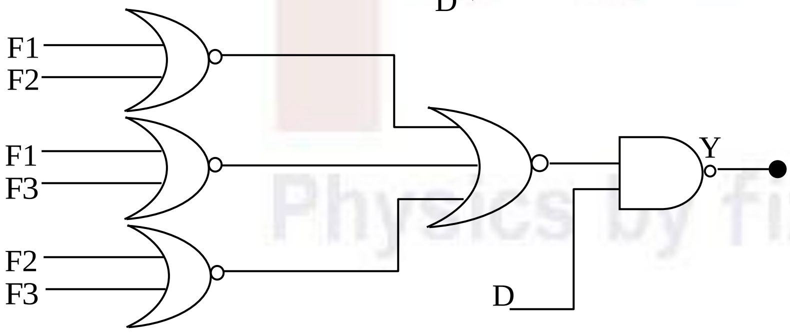
(a)



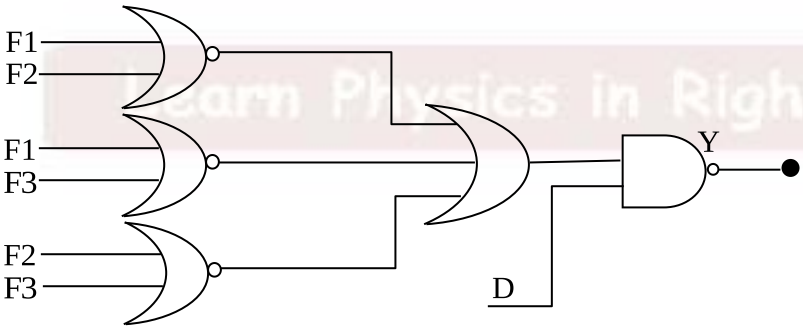
(b)



(c)



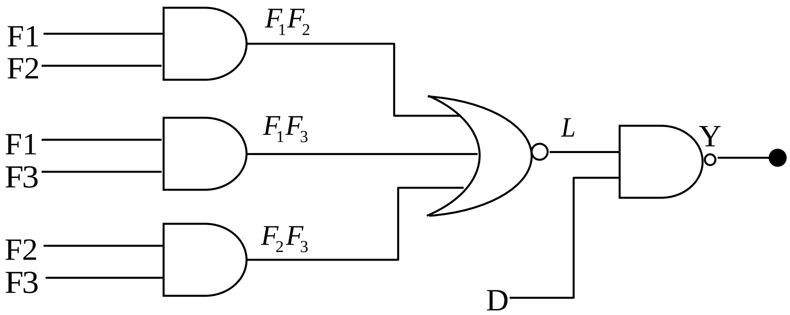
(d)



Ans. 27: option (a), (b) and (d) are possible

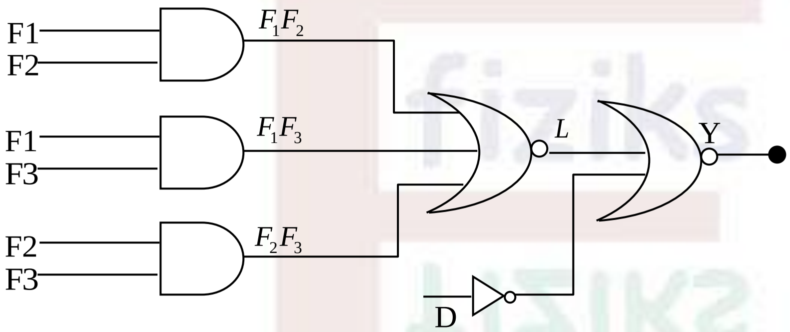
Solution.:

(a)



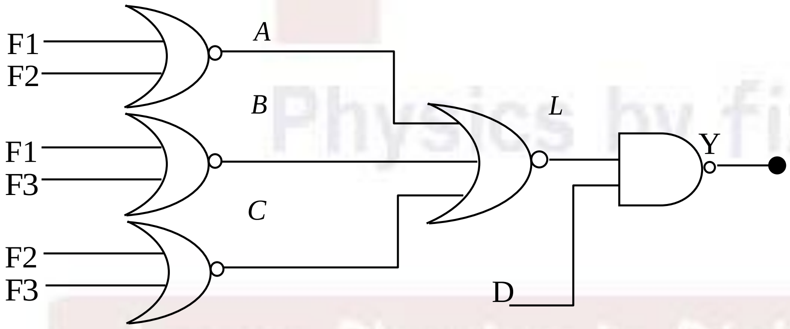
$$L=\overline{F_1F_2+F_1F_3+F_2F_3},\ D=1,\ Y=\overline{L.D}=\overline{L}+\overline{D}=\overline{\overline{F_1F_2+F_1F_3+F_2F_3}}+\overline{1}=F_1F_2+F_1F_3+F_2F_3$$

(b)



$$L=\overline{F_1F_2+F_1F_3+F_2F_3},\ D=1,\ Y=\overline{L+\overline{D}}=\overline{L}\,\overline{\overline{D}}=\left(\overline{\overline{F_1F_2+F_1F_3+F_2F_3}}\right)1=F_1F_2+F_1F_3+F_2F_3$$

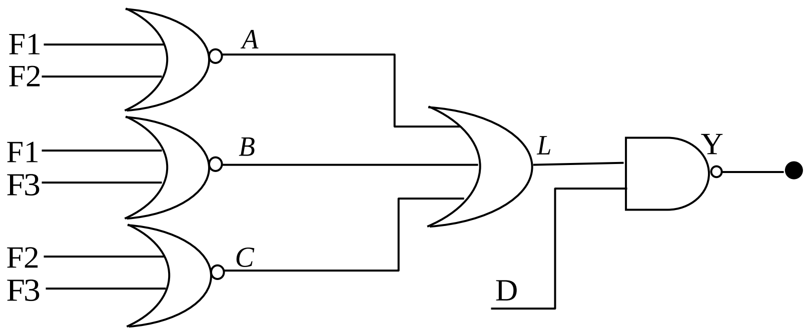
(c)



$$L=\overline{A+B+C},\ D=1,\ Y=\overline{L.D}=\overline{L}+\overline{D}=\left(\overline{\overline{A+B+C}}\right)+0=A+B+C$$

$$Y=\left(\overline{F_1+F_2}\right)+\left(\overline{F_1+F_3}\right)+\left(\overline{F_2+F_3}\right)=\overline{F_1}\,\overline{F_2}+\overline{F_1}\,\overline{F_3}+\overline{F_2}\,\overline{F_3}$$

(d)

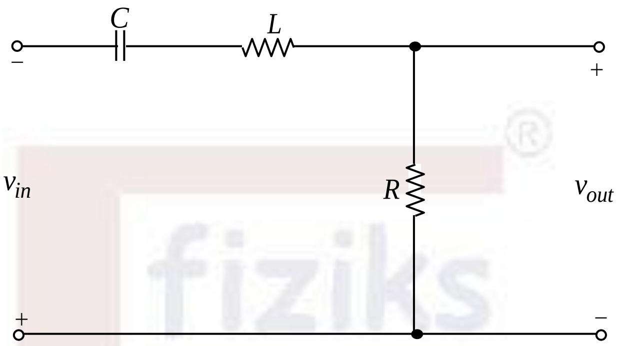


$L = A+B+C$, $D=1$ $Y=\overline{L.D}=\overline{L}+\overline{D}=\left(\overline{A+B+C}\right)+0=\overline{A+B+C}$

$$Y=\overline{\left(\overline{F_1+F_2}\right)+\left(\overline{F_1+F_3}\right)+\left(\overline{F_2+F_3}\right)}=\left(\overline{\overline{F_1+F_2}}\right)\left(\overline{\overline{F_1+F_3}}\right)\left(\overline{\overline{F_2+F_3}}\right)$$

$$\Rightarrow Y=\left(F_1+F_2\right)\left(F_1+F_3\right)\left(F_2+F_3\right)$$

Q28. In the LCR circuit shown below, the resistance $R=0.05\Omega$, the inductance $L=1H$ and the capacitance $C=0.04F$.



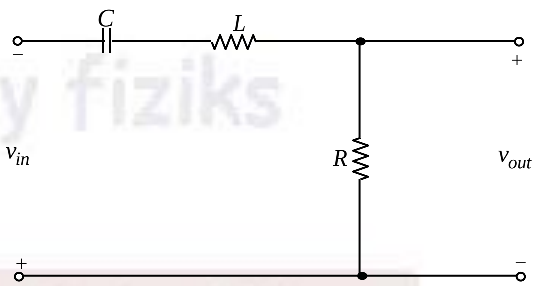
If the input v_{in} is a square wave of angular frequency $1rad/s$, the output v_{out} is best approximated by a

(a) Square wave of angular frequency $1rad/s$ (b) Sine wave of angular frequency $1rad/s$

(c) Square wave of angular frequency $5rad/s$ (d) Sine wave of angular frequency $5rad/s$

Ans. 28: (d)

Solution.: $v_{in}=1rad/s, L=1H, C=0.04F$



Resonant angular frequency

$$\omega_r=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{0.04}}=5rad/s$$

Thus, for an input frequency of $1rad/s$ (just like dc),

the LC-circuit will oscillate in sinusoidal fashion (it can only oscillate harmonically), at $5rad/s$.

Hence, (d) is the correct answer.

Q32. In an experiment, the velocity of a non-relativistic neutron is determined by measuring the time $(\sim 50ns)$ it takes to travel from the source to the detector kept at a distance L. Assume that the error in the measurement of L is negligibly small. If we want to estimate the kinetic energy T of the neutron to within 5% accuracy, i.e., $|\delta T/T|\le 0.05$, the maximum permissible error $|\delta T|$ in measuring the time of flight is nearest to

(a) $1.75ns$ (b) $0.75ns$ (c) $2.25ns$ (d) $1.25ns$

Ans. 32: (d)

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Solution:

If v is the velocity of non-relativistic neutron and t is the time taken to travel distance L

$$\therefore v = \frac{L}{t}$$

Kinetic energy $T = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{L^2}{t^2}$

Percentage error is T is $\frac{\delta T}{T} = 2\frac{\delta L}{L} + 2\frac{\delta t}{t}$

Since $\delta L = 0, \frac{\delta T}{T} = 0.09$ [maximum permissible error]

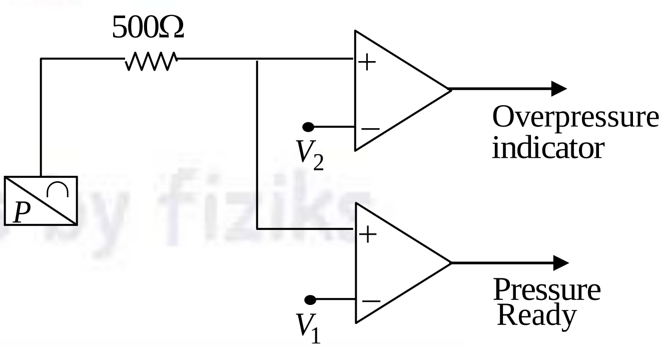
$$\therefore \frac{\delta t}{t} = \frac{1}{2}\frac{\delta T}{T} = \frac{1}{2}\times 0.05 = 0.025$$

Thus $\delta t = 0.025 \times t = 0.025 \times 50$ nsec $\Rightarrow \delta t = 1.25$ nsec

Thus, the correct option is (d)

Part C

Q55. The pressure of a gas in a vessel needs be maintained between 1.5 bar to 2.5 bar in an experiment. The vessel is fitted with a pressure transducer that generates $4\,mA$ to $20\,mA$ current for pressure in the range 1 bar to 5 bar. The current output of the transducer has a linear dependence on the pressure.



The reference voltages V_1 and V_2 in the comparators in the circuit (shown in figure above) suitable for the desired operating conditions are respectively

(a) $2V$ and $10V$ (b) $2V$ and $5V$ (c) $3V$ and $10V$ (d) $3V$ and $5V$

Ans. 55: (d)

Solution:

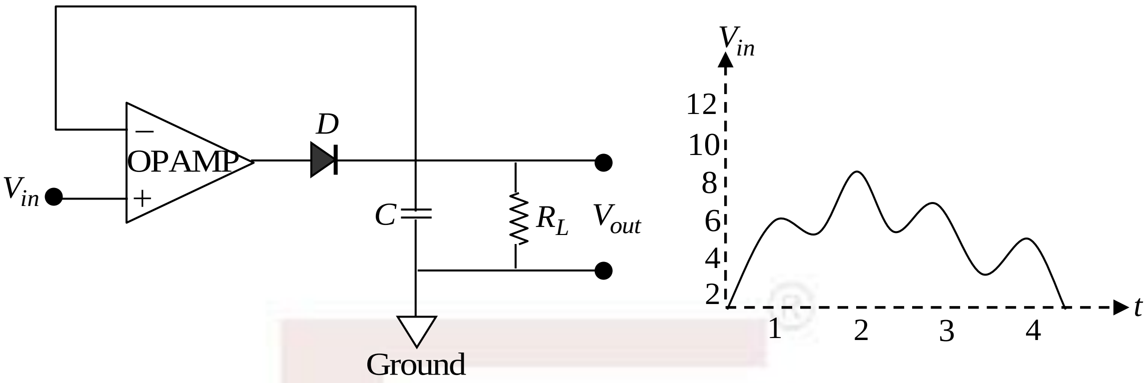
$4\,mA$ to $20\,mA$ current for pressure in the range 1 bar to 5 bar.

So 1 bar corresponds to $4\,mA$.

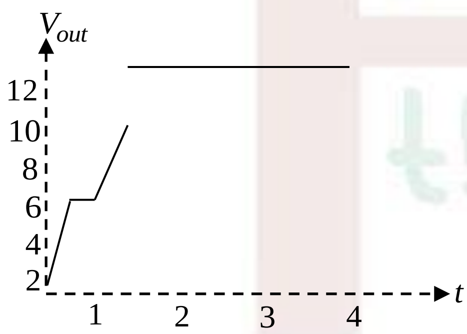
So $1.5\text{ bar} = 6\ mA \Rightarrow V_1 = 6\ mA \times 500 = 3.0\ V$

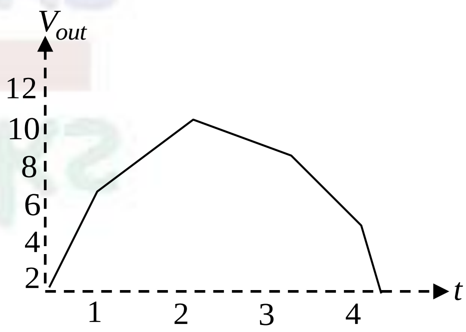
and $2.5\text{ bar} = 10\ mA \Rightarrow V_2 = 10\ mA \times 500 = 5.0\ V$

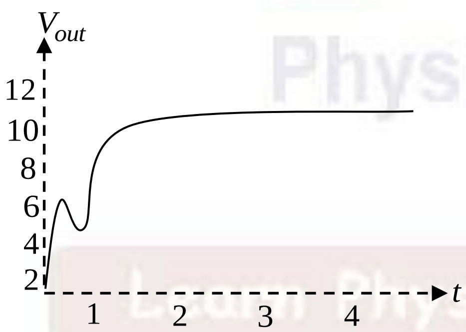
Q69. In the following circuit the input voltage V_{in} is such that $|V_{in}| < |V_{sat}|$ where V_{sat} is the saturation voltage of the op-amp (Assume that the diode is an ideal one and $R_L C$ is much larger than the duration of the measurement.)

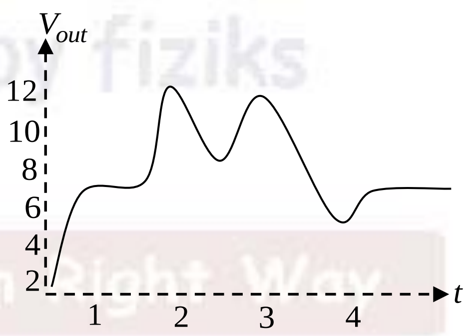


For the input voltage as shown in the figure above the output voltage V_{out} is best represented by

(a)


(b)


(c)


(d)


Ans. 69: (a)

Solution: It's a peak detector circuit so options (a) is correct.

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Solution-Atomic and Molecular Physics

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Part C

Q52. Diffuse hydrogen gas within a galaxy may be assumed to follow a Maxwell distribution at temperature $10^6\,K$, while the temperature appropriate for the H gas in the inter-galactic space, following the same distribution, may be taken to be $10^4\,K$. The ratio of thermal broadening $\Delta v_G / \Delta v_{IG}$ of the Lyman-α line from the H-atoms within the galaxy to that from the inter-galactic space is closest to

(a) 100 (b) 1/100 (c) 10 (d) 1/10

Ans. 52: (c)

Solution: The thermal broadening (or Doppler broadening) is given by $\Delta \nu_D = 1.67 \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$

Thus $\frac{\Delta \nu_G}{\Delta \nu_{IG}} = \sqrt{\frac{T_G}{T_{IG}}} = \sqrt{\frac{10^6}{10^4}} = 10$

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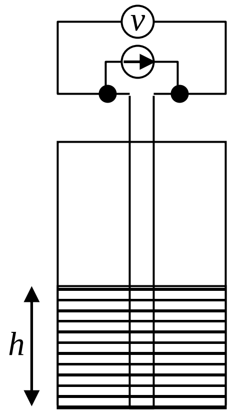
Solution-Solid State Physics

Part C

Q51. To measure the height h of a column of liquid helium in a container, a constant current I is sent through an $NbTi$ wire of length l, as shown in the figure. The normal state resistance of the $NbTi$ wire is R. If the superconducting transition temperature of $NbTi$ is $\approx 10\,K$, then the measured voltage $V(h)$ is best described by the expression



(a) $IR\left(\frac{1}{2}-\frac{2h}{l}\right)$ (b) $IR\left(1-\frac{h}{l}\right)$

(c) $IR\left(\frac{1}{2}-\frac{h}{l}\right)$ (d) $IR\left(1-\frac{2h}{l}\right)$

Ans. 51: (d)

Solution:

Since the superconducting critical temperature for NOT is 30K, the partition of the wire immersed in the liquid Helium is in the superconducting state with zero resistance, while the partition above the liquid is in normal state with resistance R where $R=\frac{\delta l}{A}$

The resistance of the wire of length l is

$$R'=\frac{\delta(l-2h)}{A}\times\frac{l}{l}=\frac{\delta l}{A}\times\frac{l-2h}{l}\Rightarrow R'=R\left(1-\frac{2h}{l}\right)$$

Since, $V=IR' \quad \Rightarrow \boxed{V=IR\left(1-\frac{2h}{l}\right)}$

Thus correct answer is (d).

Q70. Potassium chloride forms an FCC lattice, in which K and Cl occupy alternating sites. The density of KCl is $1.98\,g/cm^3$ and the atomic weights of K and Cl are 39.1 and 35.5, respectively. The angles of incidence (in degrees) for which Bragg peaks will appear when X-ray of wavelength $0.4\,nm$ is shone on a KCl crystal are

(a) $18.5, 39.4$ and 72.2 (b) 19.5 and 41.9

(c) $12.5, 25.7, 40.5$ and 60.0 (d) $13.5, 27.8, 44.5$ and 69.0

Ans. 70: (a)

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Solution:

Lattice Parameter is $a^3 = \dfrac{n_{ca} \times m}{N_A \times \delta} = \dfrac{4\times 39.1 + 4\times 35.5}{6.023\times 10^{23} \times 1.98} = 2.5\times 10^{-22}$

$a = 6.3\times 10^{-8}\,\text{cm} = 6.3A^\circ$

Bragg's law is $2d\sin\theta = \lambda \Rightarrow \sin\theta = \dfrac{\lambda}{2a}\sqrt{h^2+k^2+l^2}$

For (200) plane $\sin\theta = \dfrac{4A^\circ}{2\times 6.3A^\circ}\sqrt{2^2+0+0} = \dfrac{2}{6.3}\times 2$

$\sin\theta = 6.3175\times 2 = 6.63 \qquad \therefore \theta = \sin^{-1}(0.63) = 39.4^\circ$

Thus option (a) is correct

Q71. Lead is superconducting below $7K$ and has a critical magnetic field 800×10^{-4} tesla close to $0\,K$. At $2\,K$ the critical current that flows through a long lead wire of radius $5\,mm$ is closest to

(a) $1760\,A$ (b) $1670\,A$ (c) $1950\,A$ (d) $1840\,A$

Ans. 71: (d)

Solution: Critical field at temperature T is $B_c(T) = B_c(c)\left[1-\left(\dfrac{T}{T_c}\right)^2\right]$

Given $B_c(c) = 800\times 10^{-4}T,\ T_c = 7K$

$\therefore$ At $T = 2K$, $B_c(2K) = 800\times 10^{-4}\left[1-\left(\dfrac{2}{7}\right)^2\right]$

$B_c(2K) = 800\times 10^{-4}\left[\dfrac{49-4}{49}\right] = 800\times 10^{-4}\left(\dfrac{45}{49}\right)$

Critical current is

$$I_c = \frac{2\pi r B_c(2K)}{\mu_0} = \frac{2\cancel{\pi}\times 5\times 10^{-3}\times 800\times 10^{-4}\left(\dfrac{45}{49}\right)}{4\cancel{\pi}\times 10^{-7}} = 1837A$$

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Solution-Nuclear and Particle Physics

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Part C

Q50. The nuclei of ^{137}Cs decay by the emission of β -particles with a half-life of 30.08 years. The activity (in units of disintegrations per second or Bq) of a $1mg$ source of ^{137}Cs, prepared on January $1, 1980$, as measured on January $1, 2021$ is closest to

(a) 1.79×10^{16} (b) 1.79×10^{9} (c) 1.24×10^{16} (d) 1.24×10^{9}

Ans. 50: (d)

Solution: $A=\lambda N=\lambda N_0 e^{-\lambda t}=\dfrac{0.693}{30.08\,\text{yrs}}\times\dfrac{10^{-3}}{137}\times6.02\times10^{23}\times e^{-\frac{0.693}{30.08}\times40}$

$$=0.023\times4.3\times10^{18}\times e^{-0.922}\ \text{(Disintegration per year)}$$

$$=0.023\times4.3\times10^{18}\times0.3977=0.0393\times10^{18}\ \text{(Disintegration per year)}$$

$$=\frac{0.0393\times10^{18}}{365\times24\times60\times60}\ \text{(dps)}=1.24\times10^{9}\ \text{(dps)}$$

Q59. A ^{60}Co nucleus β-decays from its ground state with $J^P=5^+$ to a state of ^{60}Ni with $J^P=4^+$. From the angular momentum selection rules, the allowed values of the orbital angular momentum L and the total spin S of the electron-antineutrino pair are

(a) $L=0$ and $S=1$ (b) $L=1$ and $S=0$ (c) $L=0$ and $S=0$ (d) $L=1$ and $S=1$

Ans. 59: (a)

Solution.: $^{60}Co\rightarrow{}^{60}Ni+\beta^-+\bar{\upsilon}_e$

5^+ 4^+ Here $\Delta J=\pm1,\ \Delta\pi=\text{No}$

So, the given transition is allowed Gamow-Teller transition. So allowed values of the orbital angular momentum L and total spins of the electron-antineutrino pair are $L=0$ and $S=1$

Q72. The Q-value of the α-decay of ^{232}Th to the ground state of ^{228}Ra in $4082\,keV$. The maximum possible kinetic energy of the α-particle is closest to

(a) $4082\,keV$ (b) $4050\,keV$ (c) $4035\,keV$ (d) $4012\,keV$

Ans. 72: (d)

Solution: $Q_\alpha=4082\,KeV$

$$Q_\alpha=\frac{A}{A-4}K_\alpha\Rightarrow4082=\frac{232}{232-4}\times K_\alpha\ \Rightarrow K_\alpha=\frac{228}{238}\times4082=4012\,KeV$$

Q73. In the reaction $p+n \to p+K^{+}+X$ mediated by strong interaction, the baryon number B, strangeness S and the third component of isospin I_3 of the particle X are, respectively

(a) $-1,-1$ and -1 (b) $+1,-1$ and -1 (c) $+1,-2$ and $-\frac{1}{2}$ (d) $-1,-1$ and 0

Ans. 73: (b)

Solution: $p+n \to p+K^{+}+X$

	p	n	p	K^+	X
B	1	1	1	0	1
S	0	0	0	+1	–1
I_3	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	–1

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Solution-General Aptitude

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Part-A

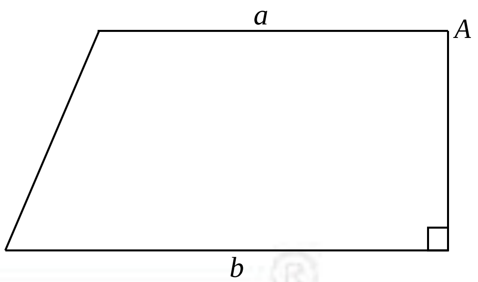
Q1. At what horizontal distance from A should a vertical line be drawn so as to divide the area of the trapezium shown in the figure into two equal parts? (a and b are lengths of the parallel sides.)

(a) $(a+b)/4$

(b) $(a+b)/3$

(c) $(a+b)/2$

(d) $(2a+b)/2$



Ans.: (a)

Solution:

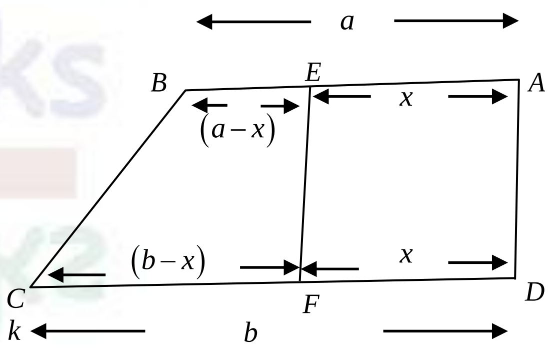
Let vertical line drawn be at a distance x from A.

$EA = x = FD$; $BE = a-x$; $CF = b-x$

Also, Area $(\square EBCF)$ = area $(AEFD)$

$$\frac{1}{2}\times EF\times(a-x+b-x) = EF\times x$$

$$\Rightarrow a+b-2x = 2x \Rightarrow 4x = a+b \Rightarrow x = \frac{a+b}{4}$$



Q2. Starting from the top of a page and pointing downward, an ant moves according to the following commands

Start

↓

Repeat actions in curly brackets 3 times

{

Repeat actions in the box 4 times

Move forward 100 units
↓
Turn right by 90 degrees

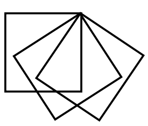
↓

Turn left by 30 degrees

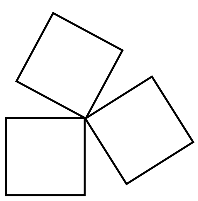
}

Of the following paths

(A)



(B)



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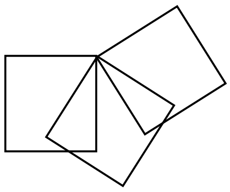
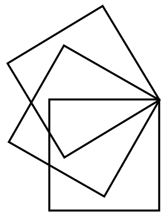
(C)



(D)



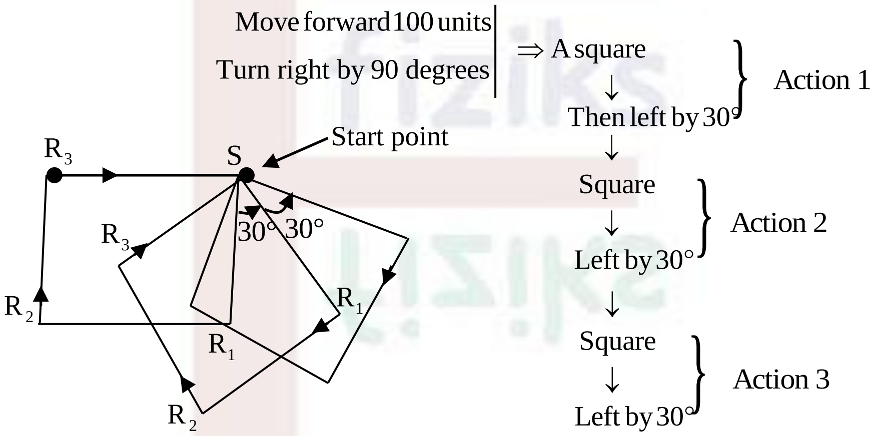
Which is the correct path of the ant?

(a) A (b) B

(c) C (d) D

Ans.: (a)

Solution:

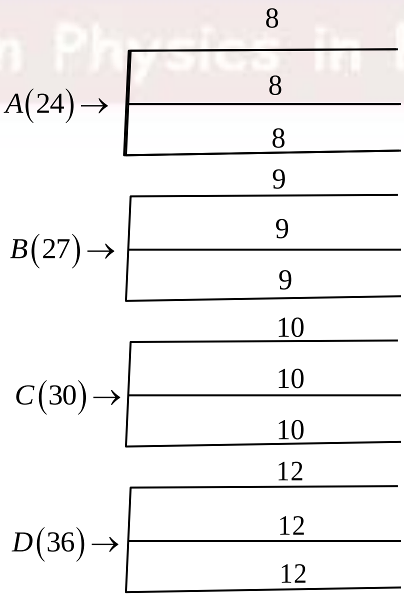


Q3. Sections *A*, *B*, *C* and *D* of a class have 24, 27, 30 and 36 students, respectively. One section has boys and girls who are seated alternately in three rows, such that the first and the last positions in each row are occupied by boys. Which section could this be?

(a) *A* (b) *B* (c) *C* (d) *D*

Ans.: (b)

Solution:



If Boys & Girls are seated alternatively and first and last position is occupied by boys (possible ways)

$BGB \to 3$; $BGBGB \to 5$; $BGBGBGB \to 7$

This is possible when number of students in each row is odd. And, only in section B, students in each row are odd in numbers.

Q4. A plant grows by 10% of its height every three months. If the plant's height today is 1 m, its height after one year is the closest to

(a) 1.10 m (b) 1.21 m (c) 1.33 m (d) 1.46 m

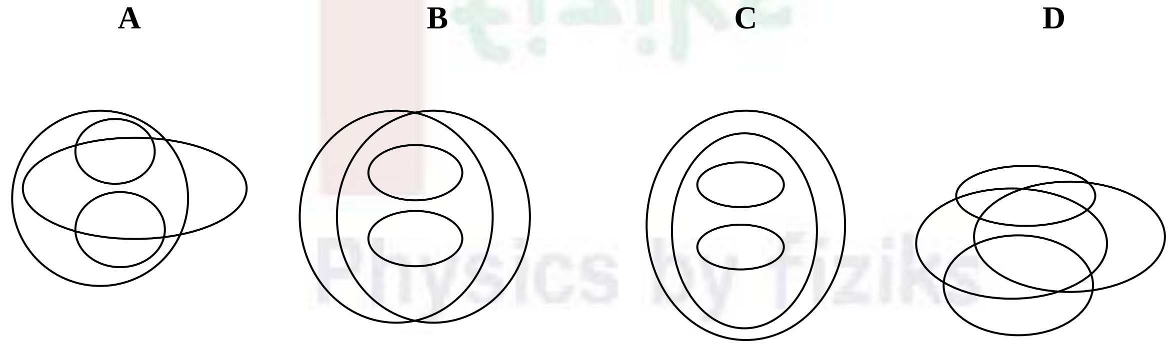
Ans.: (d)

Solution:

Let 3 months = one time span; $\therefore$ 12 months = 4 time span

$\therefore$ Growth after 4 time span $=1\times\left(1+\dfrac{10}{100}\right)^4=(1\cdot 1)^4=1.4641\text{m}\approx 1.46$

Q5. The correct pictorial representation of the relations among the categories PLAYERS, FEMALE CRICKETERS, MALE FOOTBALLERS and GRADUATES is

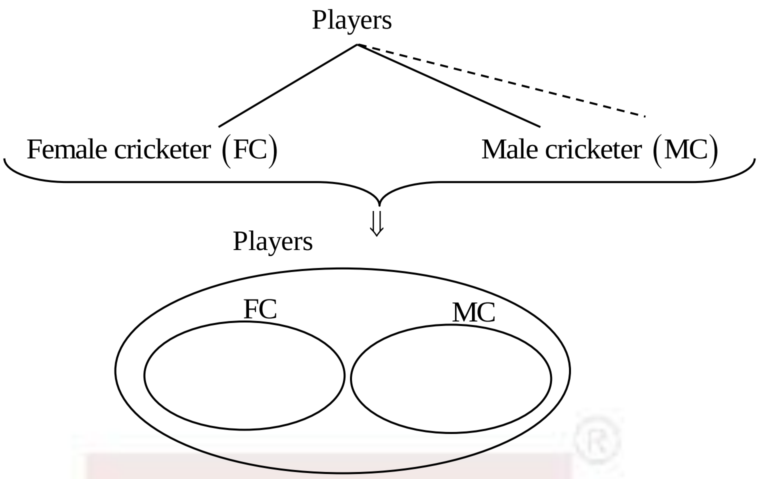


(a) A (b) B

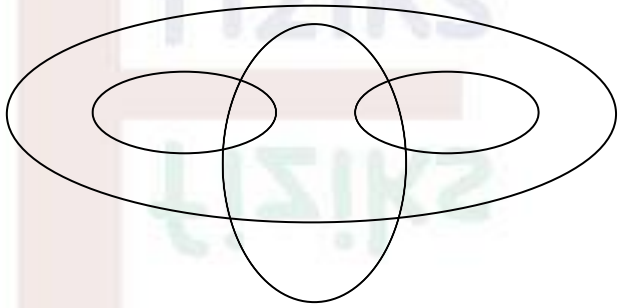
(c) C (d) D

Ans.: (a)

Solution:



Also, some players may be graduate & some not so, right choice is (a).



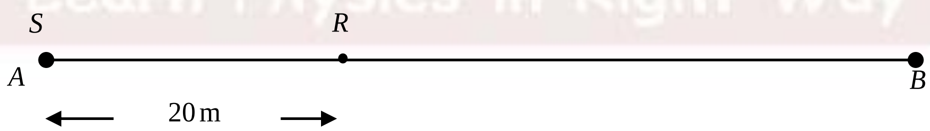
Q6. On a track of 200 m length, S runs from the starting point and R starts 20 m ahead of S at the same time. Both reach the end of the track at the same time. S runs at a uniform speed of 10 m/s. If R also runs at a uniform speed, what is R's speed (in m/s)?

(a) 9 (b) 10

(c) 12 (d) 8

Ans. : (d)

Solution:



$S = v \times t$

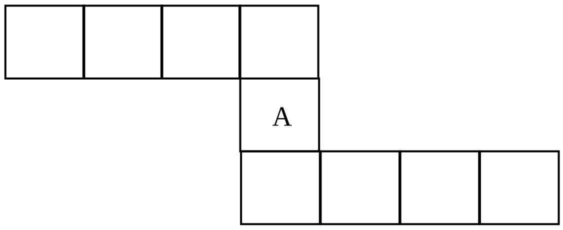
Time = const.

$\therefore$ Ratio of speeds of S and R is proportional to distance covered by them

$v_S : v_R = 100 : 80 = 5 : 4$ $\qquad \therefore \ v_R = 8\,\text{m/s}$

Distance covered by: $S = 100\,\text{m}$; $R = 80\,\text{m}$

Q7. The squares in the following sketch are filled with digits 1 to 9, without any repetition, such that the numbers in the two horizontal rows add up to 20 each. What number appears in the square labelled A in the vertical column?



(a) It cannot be ascertained in the absence of the sum of the numbers in the column

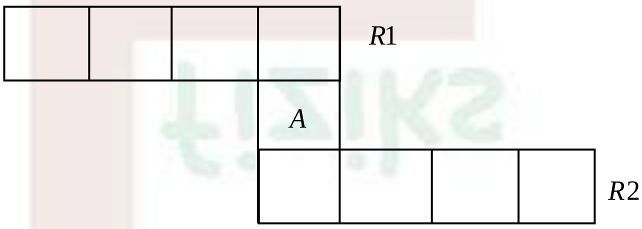
(b) 3

(c) 5

(d) 7

Ans.: (c)

Solution:



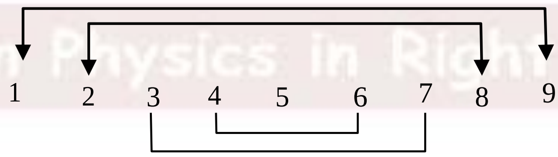
Sum of numbers in row $R_1 = 20$ and row $R_2 = 20$

$\therefore$ Sum of nos. in R_1 & $R_2 = 40$

Also, sum of all nos. from 1 to 9 = 45

$\therefore \quad A = 45 - 40 = 5$

Possible combination of nos. is Rows: $(3,4,6,7)$ & $(1,2,8,9)$.



Q8. Tokens numbered from 1 to 25 are mixed and one token is drawn randomly. What is the probability that the number on the token drawn is divisible either by 4 or by 6?

(a) 8/25 (b) 10/25 (c) 9/25 (d) 12/25

Ans.: (a)

Solution:

$n(4)$ = tokens with nos. divisible by 4 = 6.

$n(6)$ = tokens with nos. divisible by 6 = 4

$n(4\&6)$ = tokens with nos. divisible by 4 & 6 = 2 (12, 24)

$\therefore\ n(4\,\text{or}\,6) = n(4) + n(6) - n(4\,\text{and}\,6) = 6 + 4 - 2 = 8$

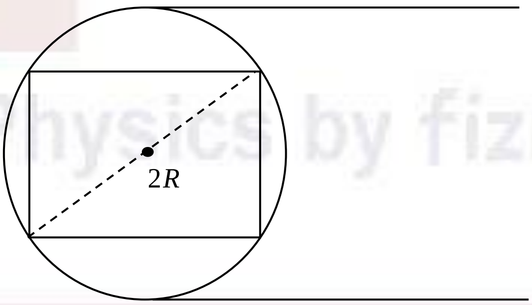
$\therefore$ Required probability $= \dfrac{8}{25}$

Q9. A beam of square cross-section is to be cut out of a wooden log. Assuming that the log is cylindrical, what approximately is the largest fraction of the wood by volume that can be fruitfully utilized as the beam?

(a) 49% (b) 64% (c) 71% (d) 81%

Ans.: (b)

Solution:



Let the radius of cylindrical log = R and length $= \ell$

$\therefore$ Largest cross-section with square as shape will have diagonal = Diameter of cylinder,

$\therefore$ Let side of square $= a$; $\therefore$ Diagonal of square $= \sqrt{2}a$

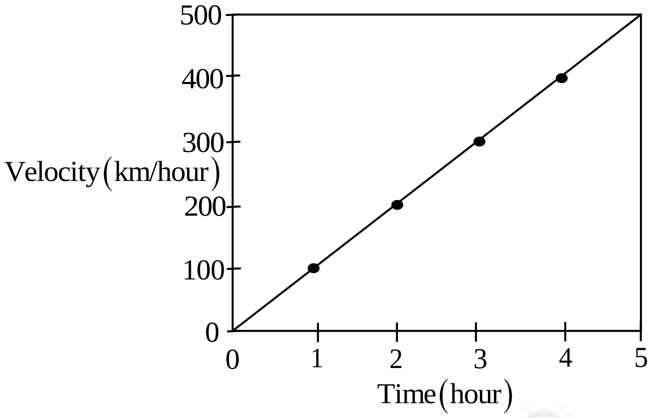
$\sqrt{2}a = 2R \Rightarrow a = \sqrt{2}R$

Fruitful largest volume $= a^2 \times \ell = \left(\sqrt{2}R\right)^2 \times \ell = 2R^2\ell$

Volume of cylinder $= \pi R^2 \ell$; $\therefore$ Fraction $= \dfrac{2R^2\ell}{\pi R^2 \ell} = \dfrac{2}{\pi}$

$y = \dfrac{2}{\pi} \times 100 = \dfrac{200}{\pi} \approx 64\%$

Q10. Given plot describes the motion of an object with time.



(a) The object is moving with a constant velocity.

(b) The object covers equal distance every hour.

(c) The object is accelerating.

(d) Velocity of the object doubles every hour.

Ans.: (c)

Solution:

Choice (a) : Not possible, object has different velocity at different time.

Choice (b): Distance covered in 1st hour

Area under curve $=\frac{1}{2}\times 1\times 100=50\ \text{km}$

Distance covered in first 2 hour $=\frac{1}{2}\times 2\times 200=200\ \text{km}$

Distance covered between 1st and 2nd hour $=200-50=150\ \text{km}$

$\therefore$ Not correct choice.

Choice (d) is also incorrect.

$\therefore$ Right choice is (c).

Q11. In a four-digit PIN, the third digit is the product of the first two digits and the fourth digit is zero. The number of such PINs is

(a) 42 (b) 41

(c) 40 (d) 39

Ans.: (a)

Solution:

$\underline{a}\ \ \underline{b}\ \ \underline{a\times b}\ \ \underline{0}$

For, $a\times b=0$

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Total nos. of pairs of $(a,b)=19$

$\underline{a}$	$\underline{b}$	$\underline{a \times b}$	
0	0	0	= 1
1	0	0	
2	0	0	
3	0	0	
4	0	0	
5	0	0	Total = 9
6	0	0	
7	0	0	
8	0	0	
9	0	0	

Being PIN, (a,b) & (b,a) are different.

$\therefore$ So, total no. of such PIN with $a \times b = 0$ is 19.

For $a \times b = 1$	Total PIN = 1	$(1,1)$;
$a \times b = 2$	Total PIN = 2	$(1,2)\&(2,1)$;
$a \times b = 3$	Total PIN = 2	$(1,3)\&(3,1)$;
$a \times b = 4$	Total PIN = 3	$(1,4),(4,1),(2,2)$;
$a \times b = 5$	Total PIN = 2	$(1,5);(5,1)$
$a \times b = 6$	Total PIN = 4	$(a,6),(6,1),(2,3),(3,2)$;
$a \times b = 7$	Total PIN = 2	
$a \times b = 8$	Total PIN = 4	$(1,8),(8,1),(2,4),(4,2)$;
$a \times b = 9$	Total PIN = 3	$(1,9),(9,1),(3,3)$

$\therefore$ Total $=19+23=42$.

Q12. I have a brother who is 4 years elder to me, and a sister who was 5 years old when my brother was born. When my sister was born, my father was 24 years old. My mother was 27 years old when I was born. How old (in years) were my father and mother, respectively, when my brother was born?

(a) 29 and 23
(b) 27 and 25
(c) 27 and 23
(d) 29 and 25

Ans.: (a)

Solution:

Let age of "Me" $=x$, Elder brother $=x+4$, Sister $=x+4+5$, Father $=x+4+5+24$

Mother $=x+27$

$\therefore$ When brother was born:

Father's age $=(x+4+5+24)-(x+4)=29$

Mother's age $=(x+27)-(x+9)=23$.

Q13. A liar always lies and a non-liar, never. If in a group of n persons seated around a round-table everyone calls his/her left neighbor a liar, then

(a) all are liars.

(b) n must be even and every alternate person is a liar

(c) n must be odd and every alternate person is a liar

(d) n must be a prime

Ans. : (b)

Solution:

Suppose (1) is non-liar (T), $\therefore 2 \rightarrow$ Liar (L)

2 tells 3 is liar, and 2 himself is liar.

This implies 3 is T.

3 tells 4 is liar, and 3 is T, this implies, 4 is liar and so on.

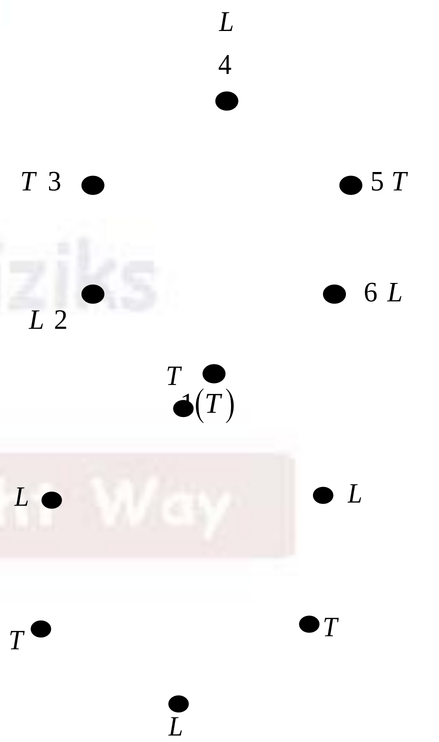
This is possible when their no. is even and every alternate is a liar.

Suppose (1) is Liar (L)

Then possible Diagram (Like above):

Again we see, the same thing.

$\therefore$ Correct choice (b).



Q14. A boy has kites of which all but 9 are red, all but 9 are yellow, all but 9 are green, and all but 9 are blue. How many kites does he have?

(a) 12
(b) 15
(c) 9
(d) 18

Ans.: (a)

Solution:

Let no. of kites $=x$, Red kites $=x-9$, Yellow kites $=x-9$, Green kite $=x-9$,

Blue kites $=x-9$

$\therefore$ Total kites: $(x-0)+(x-9)+(x-9)+(x-9)=4(x-9)$

Also, Total kites $=x$

$\therefore\ 4(x-9)=x$ or $3x=36 \Rightarrow x=12$.

Q15. If one letter each is drawn at random from the words CAUSE and EFFECT, the chance that they are the same is

(a) 1/30 (b) 1/11

(c) 1/10 (d) 2/11

Ans.: (c)

Solution:

$\underline{C}\,AU\,S\,\underline{E}$; $\underline{E}\,F\,F\,\underline{E}\,\underline{C}\,T$

Total Letters = 5 + 6 = 11

Common letters: 'C' & 'E'

Probability of pickup 'C' from CAUSE $=\frac{1}{5}$

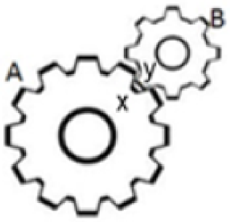
Probability of picking C from EFFECT $=\frac{1}{6}$

$\therefore$ Probability of picking 'C" from both $=\frac{1}{5}\cdot\frac{1}{6}=\frac{1}{30}$

Similarly, probability of picking 'E' from CAUSE & EFFECT is $=\frac{1}{5}\cdot\frac{2}{6}=\frac{2}{30}$

$\therefore$ Probability of picking 'C' or 'E' $=\frac{1}{30}+\frac{2}{30}=\frac{3}{30}=\frac{1}{10}$

Q16. A vehicle has tyres of diameter 1 m connected by a shaft directly to gearwheel A which meshes with gearwheel B as shown in the diagram. A has 12 teeth and B has 8. If points x on A and y on B are initially in contact, they will again be in contact after the vehicle has travelled a distance (in meters)



(a) 2π (b) 3π

(c) 4π (d) 12π

Ans.: (a)

Solution:

Point x & y will be together when A has made 2 rounds & B three rounds.

$\therefore$ Distance covered in two rounds by A: $= 2\cdot(2\pi\cdot r) = 2\cdot\left(2\pi\cdot\frac{1}{2}\right) = 2\pi$.

Q17. After 12:00:00 the hour hand and minute hand of a clock will be perpendicular to each other for the first time at

(a) 12:16:21 (b) 12:15:00

(c) 13:22:21 (d) 12:48:08

Ans.: (a)

Solution:

Let at 12 : x, minute & hour hand be at 90°.

Angle made by:

Hour hand in x min $= \frac{x^0}{2}$

Minute hand in x $= 6x^0$

$\therefore \left|6x - \frac{x}{2}\right| = 90° \Rightarrow \frac{11x}{2} = 90°$ $\qquad \therefore x = \frac{180}{11} = 16\frac{4}{11} = 16:21$

$\therefore$ They will be perpendicular at $12:16:21$.

Q18. What is the product of the number of capital letters and the number of small letters of the English alphabet in the following text?

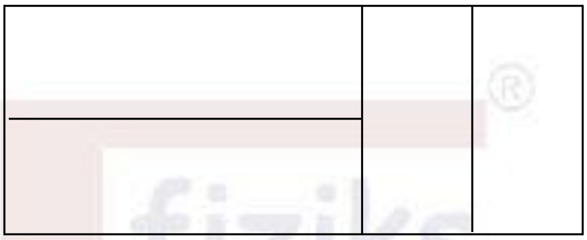
A4;={c8%$56((+B/;,.H&r]]](u];#~K@>83<??/STvx%^(d)L:/<-N347)))2;:$+}E$###[w}`”..;/89

(a) 17 (b) 37

(c) 53 (d) 63

Ans.: (d)

Q19. How many rectangles are there in the given figure?



(a) 6 (b) 7

(c) 8 (d) 9

Ans.: (c)

Q20. In a round-robin tournament, after each team has played exactly four matches, the number of wins / losses of 6 participating teams are as follows

Team	Win	Loss
A	4	0
B	0	4
C	3	1
D	2	2
E	0	4
F	3	1

Which of the two teams have certainly NOT played with each other?

(a) A and B (b) C and F

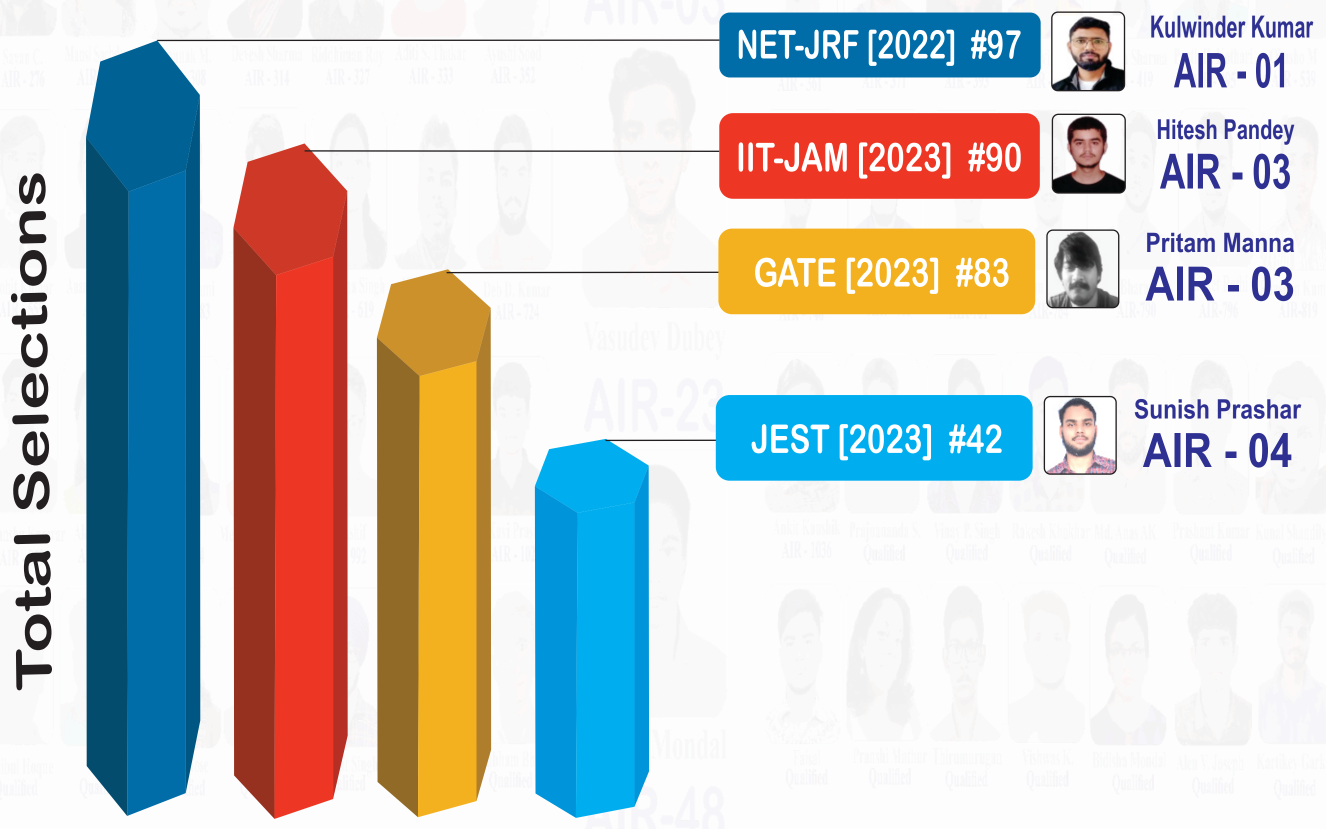
(c) E and D (d) B and E

Ans.: (d)

Solution:

B and E have containing not played with each other, had they been, at least one would have, But both are win-less.

Achivements : Year [2022-23]



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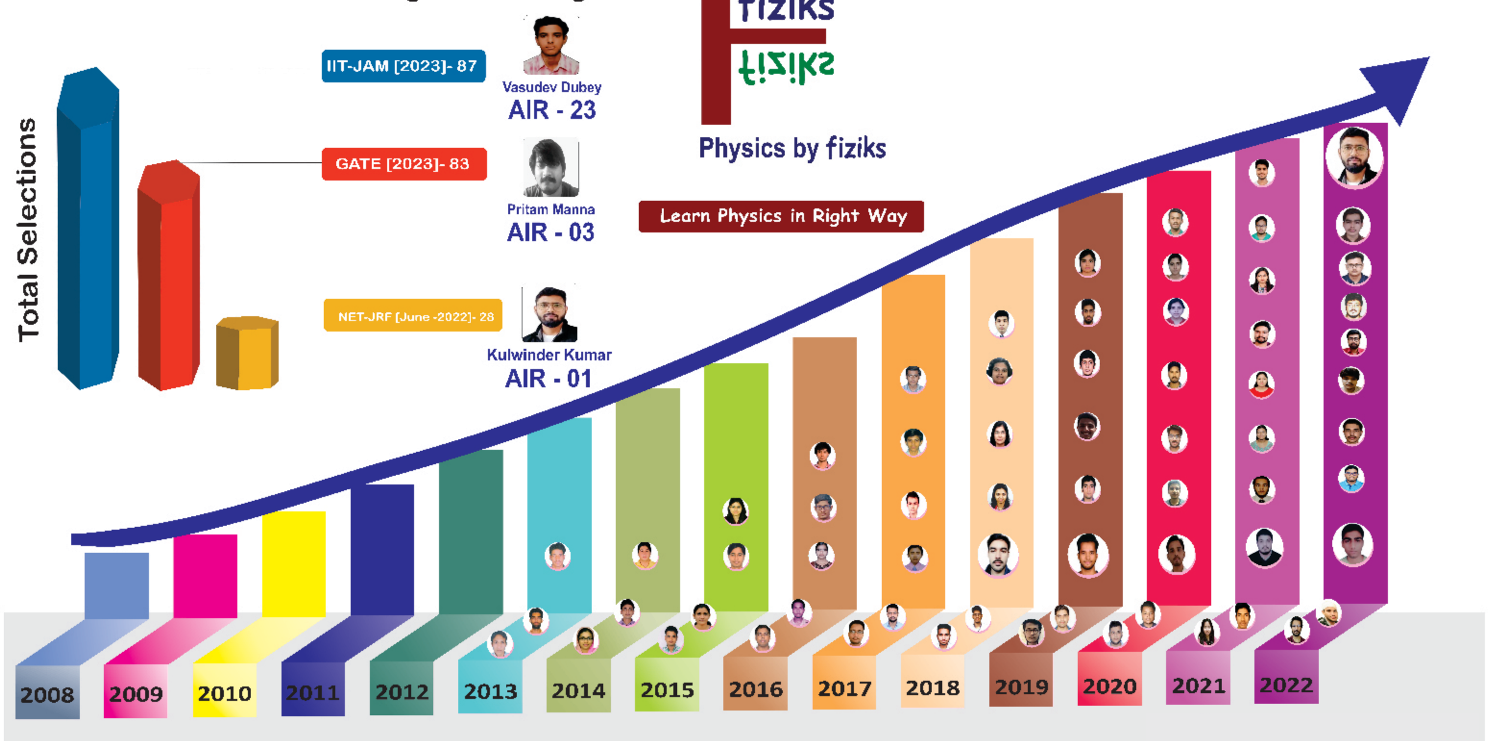

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